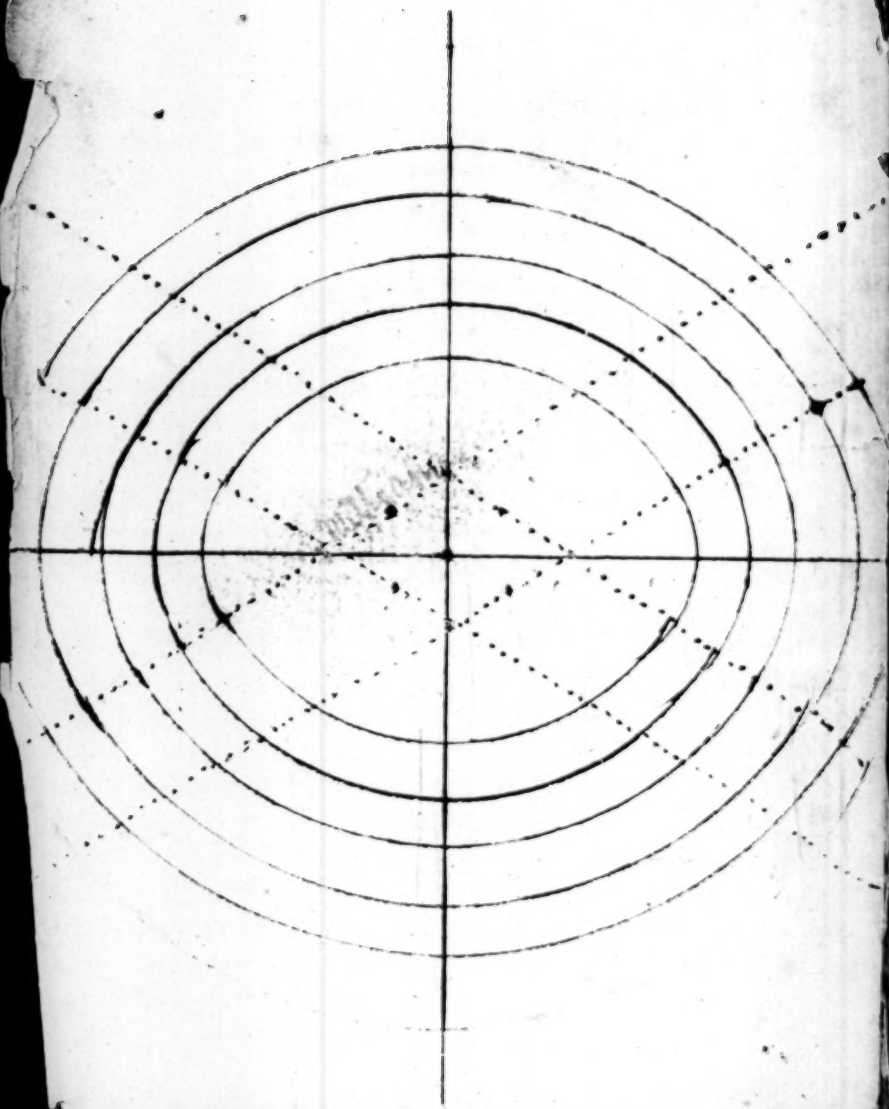
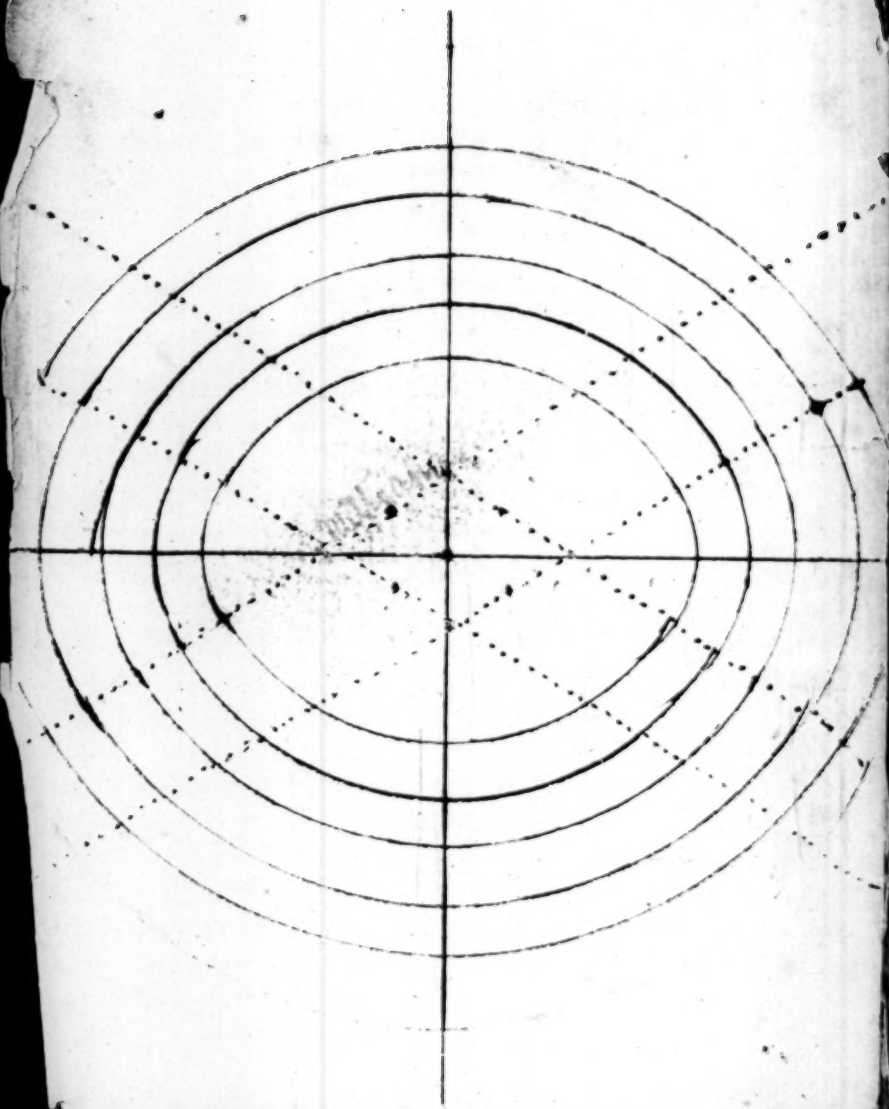


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THE
MARROW
OF THE
MATHEMATICKS,
Made Plain and Easie to the understanding
of any ordinary Capacity.

CONTAINING
The Doctrines of *Arithmetick, Geometry, Astronomy,*
Gauging, the use of the Sector, Surveying, Dyal-
ling, and the Art of Navigation, &c. Illustrated
with several Cuts for the better Explanation of
the whole Matter.

After a New, Compendious, Easy Method,
by *W. Pickering*, Merchant-Adventurer.

To which is Added,
Measuring *Surfaces and Solids*, such as *Plank,*
Timber, Stone, &c. Joiners, Carpenters, Brick-
layers, Glasiers, Painters and Paviers Work:
Each Proposition being wrought *Vulgarly, Deci-*
mally, Practically and Instrumentally.
With a Small Tract of *Gauging Wine, Ale, or Malt,*
without Inches, or Division, by which any one
may *Gauge Ten Backs, or Floors of Malt,* in
the same time another shall *Gauge One,* by the
way now used: Altogether New, and submitted
to the *Censure* of the Honourable Commissioners of
Excise.

By *J. L. P. M.*

LONDON: Printed by *T. W.* for *Eben Tracey* at
the *Three Bibles* on *London Bridge,* 1710.

THE MARRIAGE

OF THE

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TO THE
INGENIOUS
READER.

THAT which deters many from attempting, and the most from proceeding in these Studies, is, That they are wrapt up in such a Labyrinth of hard Words, and difficult Terms of Art, that a Man may be said first to learn a Language before he can come to the things themselves : And when he hath cracked the shell, and comes to the Kernel, the substance of the matter doth lye in such a Chaos of Confusion, I mean in such a scattered immethodical, and unintelligible way of handling the several parts apart, that how these do lye in the whole Body, or how one piece depends upon another, for a young beginner is extream difficult to understand, if not past finding out : Indeed

To the Ingenious Reader.

several men have writ excellently well of several things, and some have writ of all the several parts of the Mathematicks, Topically or Independently under several distinct Heads; but I have not yet met with any perfect Analysis of the whole, or such a Coherent continued Method in these Mathematical Arts as could be wished; In this small piece (therefore) I have endeavoured to remedy these inconveniencies, and supply what is defective. First, For the Terms of Art, and those affected misterious ways of expression, I have either avoided them totally, or explained them sufficiently, though whilst I endeavour to speak more plainly, perhaps by some I may be thought to speak less Learnedly; but if hereby I attain the end I aim at, I matter not; which is to be understood by any that understand Common Sense, and Plain English.

In the second place, I have endeavoured to bring the Mathematicks into one entire Method, whereby you may see how necessarily one part is Coherent and dependent upon another; and such as desire to study them, proceeding Methodically from
one

To the Ingenious Reader.

one thing to another, they may be greater Proficients in a few days, than otherwise in many Months, nay perhaps, then in all their Lives, without such a Method. Lastly, In all this, I have used so much brevity, and withal, such perspicuity, that (according to my own weak Judgment and Understanding) there is nothing inserted that could be wanting, nor any thing wanting that need be inserted: In a word, in the reading and studying many Authors, what I found excellent in any of them, I collected, and in most abbreviated, endeavouring in fewer words to make it more plain and easie to understand: And afterwards I have reduced this heap of confused Collections or Miscellanies into one perfect and entire Method, in every thing endeavouring to make the Body of the Discourse correspondent to the Title-page: Hoping that this will answer both my own and the Ingenious Readers just expectation, and shew both a nearer and clearer way to the understanding the Mathematicks: That so all may be allured by the plainness and easiness of these Arts, and Persons of ingenuity inamoured with them

for

To the Ingenious Reader.

for their usefulness and delight; and having so readily passed the Porch, and got over the Threshold, they may have both time and opportunity to build rare Superstructures upon these plain Foundations, and may improve these Arts even to miraculous Effects for the finding out many rare Inventions yet unknown to the World; whereas many (otherwise excellent wits) thorough their abstruseness have spent their whole time before they could attain to the understanding these very Rudiments. And hereupon I thought this not an unnecessary, nor an unuseful piece of work; and I doubt not but it will be acceptable to some, tho' many I know will be ready to censure both the Book and the Author; As for myself, I have been so accustomed to be misconstrued in all my Actions, that the wonder would be, not to be so in this; and indeed I do so little value it, that I can smile in disguise, to see the most so much mistaken, endeavouring in the mean time (as near as I can) to approve my self to Conscience and Omniscience. For the Book no doubt, but Carping Momus may pick some Criticisms out of it to snarl at; but I hope of no great moment;

To the Ingenious Reader.

moment ; indeed the best Actions of the best men are subject to Detraction, especially their Writings. A detractio neminem esse immunem nisi qui nihil scripserit, says a Reverend Father of our Church ; Tho. Episc. Covent. And it is well observed by another, Facillimum quidem est, & fuit semper, alienum opus reprehendere ; sed non perinde facile, aliquid simile aut præstantius conficere : And all that I shall add is that of the Poet,

Carpere vel noli nostra, vel ede tua.

Or as it may be Englished,

Either Commend me, or come and mend me.

————— Si quid novisti rectius istis,
Candidus imperti, si non, His utere
mecum.

Vale, Vive bene beateq;

T H E

To the Honorable Reader

When I first published this
little book, I was not aware
of the many errors which
it contained. Since then, I
have received many kind
communications from
gentlemen, who have
pointed out to me the
mistakes which I had
made. I have endeavored
to correct them, and I
trust that the book will
now be found to be more
correct than when it first
appeared.

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THE
P R Æ F A C E,
OR
INTRODUCTION.

THE LIBERAL ARTS or SCIENCES are reckoned Seven, *Grammar, Rhetorick and Logick, Musick, Arithmetick, Geometry, and Astronomy.* These may not unfitly be divided into Two parts:

First, The *Scholastical*, or Speaking Part, *Grammar, Rhetorick, and Logick.*

Secondly, The *Mathematical*, or Measuring Part, which comprehends *Arithmetick, Geometry, and Astronomy.*

Musick which is usually placed in the middle, partakes of a middle or mixt Nature betwixt these Two Generals. For *quoad sonum* it may belong to the former, to wit, the *Speaking Part*, being an inarticulate kind of *Speaking* : But *quoad proportionem* to
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the latter, to wit, the *Measuring Part*, consisting chiefly in due Measures and Quantities.

It is not my purpose at present to speak any thing of those which I call the *Speaking Sciences*, only to shew how proper this Division is by their Definitions you may take them thus ; *Grammatica est Ars rectè Loquendi, Rhetorica bene Loquendi, Logica bene Differendi.*

The Poet distinguishes and commends them thus ;

G. *Grata quidem Ratio est concordi voce relata.*

R. *Gratior est Ratio, veniens ratione venusta.*

L. *Grata ter est Ratio veniens ratione probata.*

Of Grammars there is as great Variety as of Languages. *Latin* is a general Language in these Western Parts of the World, and the ground of most other Languages in these parts : so is *Greek* in the Eastern : *Hebrew* the Language we shall speak in Heaven say some, and the Original Language of all others say most ; and *High Dutch* as Ancient as the Building of *Babel*, say many.

But

But of all other Grammars and Languages most suitable to this present Subject, is that most incomparably ingenious Invention of Bishop *Wilkins* (that Famous *Mathematician*) which he calls his *Philosophical Language*, with a way of writing the same, called his *Universal Character*, which if it would be as Universally, and as Industriously practised, as it is most excellently, and ingeniously contrived: By this means all Persons (at least all of any competent capacity) of what Nation or Language soever, might converse and correspond with one another as familiarly as *Adam* and *Eve* in Paradise, or those Eight in the Ark, when the whole World spoke but one and the self-same Language. But enough of these as to my purpose (to wit) of *Grammar*, *Rhetorick*, and *Logick*.

And for *Musick*, which I call the Meane in this Division, I mean not to run any large Divisions upon it; that Art being rather learn'd by Practice than by Precept; and my Aim being rather to propound Demonstration to the Eye than Musick to the Ear. Indeed the Musick of the Spheres might have been no improper Theam, but that being grounded upon the Old Erronious Opinion of their *solid Orbs*, are both disclaimed long since by later Writers. Indeed, I read that *Philo* the Jew says, that hereby *Moses* was enabled to carry in the

B 2 Mounc

Mount 40 days, and 40 Nights, without eating or drinking any thing, because he heard the melody of the Heavens : and *Plutarch* speaks as if a Man might hear this *Musick* if he were an Inhabitant in the Moon : but these conceits being by the Learned long since exploded, I shall no further touch upon this string, *viz.* *Musick*, with the other *Speaking Arts* before-mentioned : But passing by all these, That which I call the *Mathematical*, or *Measuring Part* of the *Liberal Sciences* is the subject I intend to treat upon ; and (as I apprehend it) may fitly be comprehended in these Three, *Arithmetick*, *Geometry*, and *Astronomy* : *Astronomy* may be said to measure the Heavens, *Geometry* the Earth, and both by means and help of *Arithmetick*. This last consisting chiefly in *The Rule of Three*, the two former in the Resolution of a Triangle. And when I say *Geometry* measures the Earth, it is to be understood the whole *Globe* of the Earth, containing *Sea* and *Land*, with all other Bodies therein contained, whether *Superficies* or *Solids* : What concerns the Land, may in one word be called *Surveying* : What concerns the *Sea*, is properly called *Navigation* : And not only both these, but also *Astronomy* with these may be called the Taking of Heights and Distances : Surveying being Taking Heights and Distances at Land, Navigation at Sea, Astro.

The Introduction.

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Astronomy in the Heavens, and of Astronomy, the most Mathematical and Practical part is Dialling, or finding the hour of the day by the Sun, and the hour of the night by the Moon and Stars.

Therefore to sum up all in short, the Mathematical part of the Liberal Sciences are three, viz. *Arithmetick, Geometry, and Astronomy*. The Practical part of these are principally three, *Surveying, Navigation, and Dialling*. These three are performed by Trigonometry, or the Resolution of a Triangle, which derives its name from Three, and consists of Three Angles, and Three Sides, and these found by the *Rule of Three*, makes good that old Motto, *Tria sunt omnia*.

You see then, That the grand Design of the Mathematicks, is to take Heights and Distances at Sea, at Land, and in the Heavens, and to measure not only the Heavens and the Earth, but all other Bodies therein contained, whether Superficies or Solids. At Land, to find Heights by Distances, and Distances by Heights, where either is accessible; and both these by Angles of Position and Interfection of Lines, where both or either of them are inaccessible. At Sea for Distances, to find the Distance of a Ship North and South, which is called Latitude, or its Distance East and West, which is called Longitude, or the Distance

from any of these Cardinal points, which is called the Rhumb; Lastly, the Distance Run upon the said Rhumb, or upon such a point of the Compass: I am not ignorant in strict propriety of Terms, a Rhumb differs from a point of the Compass. A Rhumb containing two points, *viz.* the Point and its opposite Point: the former being the whole Diameter, the latter only the Semidiameter of the Compass from the Center, so that there are (to speak according to that nicety) only 10 Rhumbs, and 32 Points of the Compass; but for more plainness and ease to the Learner, I make them here signifie the same thing, being the same in substance. And by an Account kept of these to know the Distance of one Ship from another, or of the same Ship from the place from whence it set Sail, or from the place to which it is bound; and consequently how to direct her course accordingly. In the Heavens, to find the Distances of the Sun, Moon, and Stars from one another, of one Star from another; particularly concerning the Sun, his Distance from the Equinoctial, which is called his Declination: His Distance from any of the 12 Signs which is called his place in the Zodiack: His Distance of Rising and Setting from the due point of East or West, which is called his Amplitude: His Distance at any time from any point of the Horizon,

zon, which is called his Azimuth ; together with the Distances of all those Coelestial Bodies, to wit the Planets, the Sun, Moon, and fixed Stars from this Terrestrial Globe : All these may be called Distances in Astronomy : And for Heights, both in Astronomy and Navigation, they are the same ; as taking the Height or Altitude of the Sun and Stars at any time above the Horizon which is called their Almicanaras, particularly the Meridian Altitude, either of the Sun or Stars : And thereby to find the Height or Elevation of the Pole : So that by all this you see, that the taking of Heights and Distances at Sea, at Land, and in the Heavens, is the most principal, and the most practical part of the Mathematicks : And these are all perform'd by Trigonometry, or by the making and measuring, the framing and resolving of a Triangle, either Plain or Sphaerical : Supposing these Heights and Distances to be the several parts, that is, either the sides or Angles of a Triangle. And to find by some of these Parts known, other Parts unknown *or per Data Quaesita.*

This Resolving of a Triangle, or Trigonometry, resolves all Mathematical Questions, and may be performed by Instruments and Protraction Geometrically : But principally by the Rule of Proportion Arithmetically ; worthily carrying the name

of the *Golden Rule*, for the admirable Effects it produceth, especially in these Mathematical Operations. And for the fuller and more ample Explanation of these particulars, I shall proceed to Discourse of them in this Method in three Books.

I. In the First I shall Treat of Arithmetick, under the several kinds of it, *viz.* *Common, Decimal, Logarithmetical, and Instrumental.* And in the sequel of the Discourse, as occasion shall be, I shall shew how these may be applied to the Resolution of a Triangle, or any other Mathematical Question.

II. In the Second Book I shall Treat of Geometry, or any other Mathematical Question. First in general, and of the Three Famous Grametrical Figures, the *Circle, Square, and Triangle*; especially the last of these. And then Secondly, in particular I shall speak of Geometry, as it consists in Surveying and Navigation; and shew that these are no more than the Resolution of a plain Triangle.

III. In the third and last Book I shall Treat of Astronomy, and therein of Projection of the Sphere, Questions of the Sphere and Dialling, and shew that as Surveying and Navigation are performed by the Resolution of a Plain Triangle, so these by the resolution of a Spherical Triangle.

The First BOOK.

O F

ARITHMETICK.

C H A P. I.

Which as before I do divide into 4 Parts.

1. Common; 2. Decimal, 3. Logarithmetical, 4. Instrumental, or Lineal. *Of these in their order.*

COMMON ARITHMETICK to those that are expert in it, especially in working by Fractions, is of singular use, and absolutely necessary; and for the most part of sufficiency enough for Mathematical Operations: and withal it is to be supposed, That no Man that attempts the Mathematicks, but in some measure understands those Common Things of *Numeration, Addition,*

Subtraction, Multiplication, and Division, therefore I shall be very brief in them ; and only tell you.

S E C T. I.

Numeration is the Reading of Figures according to the value they receive from the place they stand in : The First place signifying Units : the Second Tens : the Third Hundreds : And the Fourth Thousands, according to the ensuing Table.

Numeration

Numeration TABLE.

Units.	1
Tens.	12
Hundreds.	123
Thousands.	1234
X. Thousands.	12345
C. Thousands.	123456
Millions.	1234567
X. Millions.	12345678
C. Millions.	123456789
Thousand Millions.	1234567891
X. Thous. Millions.	12345678912
C. Thous. Millions.	123456789123
Dillions.	1234567891234
X. Dillions.	12345678912345
C. Dillions.	123456789123456
Thousand Dillions.	1234567891234567
X. Thous. Dillions.	12345678912345678
C. Thous. Dillions.	123456789123456789
Trollions.	1234567891234567891
X. Trollions.	12345678912345678912
C. Trollions.	123456789123456789123
Thousand Trollions.	1234567891234567891234
X. Thous. Trollions.	12345678912345678912345
C. Thous. Trollions.	123456789123456789123456

One Hundred Twenty Three Thousand
Four Hundred Fifty Six Trollions Seven
Hundred Eighty Nine Thousand One Hun-
dred Twenty Three Dillions Four Hundred
Fifty Six Thousand Seven Hundred Eighty
Nine Millions One Hundred Twenty Three
Thousand Four Hundred Fifty Six.

SECT.

S E C T 2.

II. *Addition* is the adding of two or more Sums together, to find the Total.

In like Denominations. In divers Denom.

		D.	M.	Sec.
6789432	{ Degrees D. }	136	11	9
1903467	{ Minutes }	320	10	45
8542079	{ Seconds }	429	54	26
17234978	Summa Total.	886	16	20

S E C T 3.

III. *Subtraction* is the Subtracting or Deducting of one Sum from another, to find the Remainder.

	D.	M.	Sec.
As from	2345	36	49
Take	873	18	13

Remainder is 1472 18 36

S E C T.

S E C T. 4.

IV. *Multiplication* is a Compendious *Addition*.

The TABLE.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Find the Multiplicand in the Top of the TABLE, and the Multiplier in the side thereof, and in the Square where These meet, is the Product of the Nine Digits; as 7 times 9 is 63, and so of the rest.

In Multiplication the Sum to be Multiplied is called the Multiplicand, the Sum by which the other is Multiplied, is called the Multiplier, and the Number produced by these is called the Product.

Multipli-

Multiplicand	463298765
Multiplier	23

	1389896295
	926597530

Product	10655871595
---------	-------------

To Multiply compendiously by any of the Nine Digits, without charging the Memory.

To Multiply any Number by 2,
Either double it, or set it down twice,
and add the two Sums together.

To Multiply by 3,
To the Number given, add the double
thereof, the Sum of them shall be the Pro-
duct.

To Multiply by 4,
Double the Duplication of the Sum given.

To Multiply by 5,
To the Number given add a Cypher,
and the half of that Number is the Product.

To Multiply by 6,
To the Number given add a Cypher,
then take the half of that Number, and to

I.
65
23
—
95
—
5
—
e
BOOK I. *Of Arithmetick.*

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it add the Number given, the Sum of them shall be the Product.

To Multiply by 7,

To the Number given add a Cypher, and take the half thereof, to which half add the double of the Number given, the Sum shall be the Product.

To Multiply by 8,

To the given Number add a Cypher, and from thence subtract the double of the Number given, the Remainder will be the Product.

To Multiply by 9,

Add a Cypher to the given Number, and from thence subtract the given Number, the Remainder shall be the Product.

To Multiply by 10 add a Cipher, to multiply by 100 add two Cyphers, by 1000 add three Cyphers, and it is done.

If Cyphers be in the middle of the Multiplier among other Figures, remove the next Figure a place farther where any Cypher comes, and add the Lines together in the same order they stand.

S E C T.

S E C T. 5.

Division is a compendious Subtraction, wherein the Sum to be divided is called the *Dividend*; the Sum by which we divide the same is called the *Divisor*; what is produced thereby is called the *Quotient* and the *Fraction*. *Example.*

$$\begin{array}{r}
 2 \times 1 \\
 109 \times 8 \\
 \text{Dividend } 4679138 \text{ } [203440 \frac{2}{3} \text{ parts.} \\
 \text{Divisor } 2333333 \text{ } \text{Quot. Fraction.} \\
 22222
 \end{array}$$

In dividing any great Sum, to know certainly what Figure to put in the Quotient, and to Divide the Sum by Subtraction instead of Multiplication.

First, Set down your Divisor, and against it the Digit 1, in the next place double it, and against that set the Number 2. Add the former two Sums together, and against that set the Figure 3, to that Number add the Divisor, and against that set the Figure 4, &c. so you will have a Table of the 9 Digits, and may see against every Figure how much the Multiplication of your Divisor by any Digit will amount unto; so that when you see your Remainder

in

in your Division, look in the Table for the nearest Number less than your Remainder, and that Digit which stands against that Number, you must always put in your Quotient.

To Divide by any Number that hath Cyphers to the Right Hand.

To Divide by 10, cut off the last Figure towards the Right Hand, and the rest of the Figures are the Quotient. To Divide by 100 cut off two of the last Figures, by 1000 three, &c. To Divide by 20 cut off the last Figure, and take the half of the Remainder.

As for Example,

To know how many Pounds there is in 243[6 Shillings is the same as to Divide by 20. Therefore cut off the last Figure 6, and take half the rest, that is, half 243 is 121 Pounds, and there remains 1, and the 6 cut off for the Fraction, which is $\frac{1}{2} \frac{6}{10}$, or 16 shillings, and so in any other Sum. The same Reason holds if you be to divide by 30 cut off the last Figure, and take the third part; and if by 40 the Fourth, 50 the Fifth part, and so forwards. The remaining Fraction being always the parts of its respective Integer.

These

These Four, viz. *Addition*, *Subtraction*, *Multiplication*, and *Division* (not to mention *Numeration*, which is only the Reading of Figures) do contain the whole Practice of Arithmetick, being judiciously applyed and made use of according to the Nature of each several Question, and by the Invention of Logarithmes these Four are reduced to these Two only, viz. *Addition* and *Subtraction*, as you will hear hereafter.

But besides these Essential parts of *Arithmetick*, there are divers other necessary Rules for directing the way or manner of Operation by these.

S E C T. 6.

Therefore in the sixth place is that incomparable *Rule of Three*, worthily carrying the Name of the *Golden Rule*, especially, and above the rest to be taken notice of; which yet in the Operative and Practick part of it, is no more but the right Application of Multiplication and Division, that is, by Three Numbers given, to find a Fourth, according to these Two Directions.

First, If the Third Number be greater than the First, and the Fourth required to be greater than the Second, then Multiply the Second by the Third, and Divide the
Pro-

BOOK I. Of Arithmetick.

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Product by the First, the Quorient gives the Fourth Number.

As if 12 yds of Cloth cost 3 l. what cost 435 yds.

$$\begin{array}{r} 3 \\ \hline 1305 \end{array} \quad \begin{array}{r} 12(9 \\ 1305 \overline{) 11081\frac{2}{3}} \text{ or } 1\frac{1}{4} \text{ l.} \\ 1222 \\ 11 \end{array}$$

Secondly, If the Third Number be greater than the First, and the Fourth required to be less than the Second, then Multiply the Second by the First, and Divide by the Third. As,

If 12 Men raise a Fort in 8 days, in how many days shall 24 Men do it.

$$\begin{array}{r} 8 \\ 12 \\ \hline 96 \end{array} \quad \begin{array}{r} 18 \\ 96 \overline{) 1728} \\ 24 \end{array}$$

I. The Direct Rule is, when more requires more, and less less.

II. The Back Rule is, when more requires less, or less more.

For the placing the Numbers right, observe this Rule in the *Direct Rule of Three*.

Three Numbers being given, the Question is annexed but to One, and this must always

always be the Third Number, and that which agrees with this Third in the same Denomination must be the First, and that which remains the Second. As if the Question were put, If 10 yards cost 8*l*. how many yards may we buy for 12*l* place them thus; If 8 buy 10, what will 12 buy.

S E C T. 7.

The next Rule which I shall here mention, is the *Rule of Fellowship*, which is, when several Merchants or other Men put in several Sums together into Stock, and Trading therewith, they either gain or lose so much in the whole; Now the Question will then be, what every particular Adventurer should have of the Profit, or bear of the Loss according to each mans due proportion. To do this, say, If the whole Stock produce so much as is the whole Profit or Loss, then what gives the First Man, Second and Third man's part of Stock severally making each Man's particular Stock the Third Number to these other Two Numbers, (*viz.* the whole Stock, and the whole Profit or Loss) and the Fourth will be each particular Man's Profit and Loss.

As if Three Merchants put into Stock 1684 Pounds, The First puts in 750*l*. the Second 620*l*. and the Third 314*l*. They
Trade

Trade together till they gain 500 *l.* The Question is what every one ought to have for his share.

For the First say,

If 1684 *l.* give 500 *l.* what gives 750 *l.*

For the Second say,

If 1684 *l.* give 500 *l.* what gives 620 *l.*

For the Third say,

If 1684 *l.* give 500 *l.* what gives 314 *l.*

S E C T. 8.

To find a mean proportional, Multiply one Extream Number by the other, and the Square Root of the Product is the mean proportional: As 72 multiplied by 32, the Product is 2304, the Square Root whereof is 48: For proof 48 multiplied by 48, produces the same Number 2304, and this is the mean proportional betwixt 72 and 32.

S E C T. 9.

To conclude this particular, The Extracting the Square and Cube Root is the hardest Lesson in *Common Arithmetick*: Those which have a mind to do them this way, I shall refer them to the Sea-man's *Kalendar*, in the latter end of which Book it is plainly set

set down : But this way being very difficult and tedious, I shall shew you very short and easie ways hereafter to do it by *Logarithmes*, *Gunters Sector*, *Wingates Rule of Proportion*, or any ordinary Line of Numbers, whereunto I refer the Reader at this time, and shall proceed to the Rule of Falschood.

S E C T. 10.

The *Rule of Falschood* is so called, because it teacheth (by feigned Numbers taken at all adventures) to find out the true Number that is demanded; and that 1. Either by one False Position : Or, 2. When it requires two False Positions.

The Rule of one false Position is this, instead of 3. Numbers given in the *Rule of Three*, by those to find a fourth : In this Rule we have but one Number that cometh in use to work by : Unto the likeness whereof we must feign or devise two other Numbers, which shall hold the same proportion to one another which the thid number given shall have to the number sought. As for Example ; I delivered at Interest a certain Sum of Money at 6 per Cent. and at the end of 10 years I received 500 *l.* for Principal and Interest, I demand how much was the Principal Sum ?

To answer this, let us feign a Number at pleasure, and with the same let us make

our

our Discourse even as if it were the principal Sum we seek for: As suppose I delivered him at the first 200 *l.* which in 10 years at 6 per Cent. *Simple Interest* will amount to 120 *l.* which added to the principal 200 *l.* makes together 320 *l.* but this should be 500 *l.* Therefore say by the *Rule of Three*, if 320 *l.* come of 200 *l.* what doth 500 *l.* come of? Multiply the two last Numbers 200 and 500 *l.* by one another, they amount to 10000 *l.* which divide by the first Number 320 and the Quotient is $312\frac{1}{2}$, which is the Sum sought for, or the Principal Sum delivered at Interest at the first. You will find the same Number, if you suppose 160 *l.* to come of 120 *l.* then 500 *l.* will in like manner come of $312\frac{1}{2}$ as before: Or any other supposed Numbers which hold the same proportion to one another, which 500 *l.* holds to the Number sought.

2. The Rule of Two False Positions I shall endeavour to make plain on this manner, suppose an 100 Ducats is to be divided amongst 3 Persons, the first to have a certain number of them, the second must have twice so many as the first, abating 8 Ducats, and the third must have 3 times so many as the first, less by 15 Ducats. Now I demand how many every one should have.

First, You may imagine any number at pleasure which you shall call the first Position,

tion, and with the same you shall work instead of the true Number; and if you see you have missed of the true Number that you seek, then is the last number of the work either too great, or too little; which number you shall note with the sign of more or less, and that you call the first Error; now the sign of more shall be noted with this Figure $+$, the sign of less with this plain Line $-$. For Example, Suppose the first man in the Question had 30 Ducats, then by the order of the Question, the second should have 52, the third 75, these three added make 157, and I should have but 100; so that this first Error is too much by 57, and therefore I set down the first Position 30 with his Error 57 thus, $30+57$.

The next thing to be done is to suppose another Number, which you must call your Second Position, and note its excess or want of the true number, and that you call the second Error. As for Example, Suppose you take 24 for the first Man in the Question, then by the order of the Question, the second was to have 40, and the third 57, these three added make 121, and I must but have 100, so the second Error is too much by 21, and therefore I note it down under the former thus $\left. \begin{array}{l} 30+57 \\ 24+21 \end{array} \right\}$ Then

you must Multiply the first Position 30 by the second Error 21 Crosswise ; and again the second Position 24 by the first Error 57, (and this must always be observed) and note down the two Products, viz. 630, and 1368, then observe this Rule.

*Both signs alike Subtraction do require,
But unlike signs Addition will desire.*

And accordingly, if both signs be alike, that is, both too much, or both too little, you must subtract the lesser Product from the greater ; as in this Example, 630 from 1368, and note the Remainder, which is 738, and this must be your Dividend. Again, you must subtract the lesser Error from the greater, viz 21 from 57 Resis 36, this must be your Divisor. Therefore Divide 738 by 36, the Quotient shall be the true Number sought, or the Number of Ducats which the first man was to have, viz. $20\frac{1}{2}$, and consequently the second man had 33, and the third $46\frac{1}{2}$, making in the whole 100.

But note, that if the signs of the two Errors had been unlike, that is, if one of the Errors had been too much, and the other Error too little, then instead of subtracting the one Product from the other, you must have added the one to the other ; and likewise you must have added both the

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Errors

Errors together, and by the Sum of those Errors you must have divided the Total Sum of both the Products, and the Quotient should then have been the true Number sought. As for Example.

If the two Positions were 24 and 20, you will find the first Error will be 21 too much, and the second will be 3 too little. Therefore multiply 24 by 3, produceth 72. Likewise, multiply 20 by 21 produceth 420, and because the signs of the Errors are unlike, add them 492, which shall be your Dividend; and again add the lesser Error 3 with the greater 21, makes 24 for your Divisor; and lastly, Divide 492 by 24, the Quotient is $20\frac{1}{2}$ as before, for the first man's share; and consequently the second must have 33, and the third $46\frac{1}{2}$, making up together 100, as in the former Example. And this I hope is sufficient for the understanding of this excellent *Rule of Falshood*, or of *False Positions*, in the several kinds of it; though I might have added variety of other Examples: But to avoid prolixities, and for brevities sake these may suffice.

C H A P. II.

Of Decimal Arithmetick.

IN the Second place most Mathematicians do use and extol *Decimal Arithmetick*, and do accommodate all their Scales, Rules, and Lines to this kind of *Division*, and not without cause, whereby they have this advantage, that the Fractions which in Common Arithmetick are of divers Denominations, in this are all Decimal Fractions; that is, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$ parts of any thing, and by adding Cyphers, they may bring them to smaller Fractions, and this makes the working by Fractions far more easie and exact.

To Reduce ordinary Fractions into Decimal.

Multiply the Numerator (or the Number above the line) by 100, or 1000, and Divide the Product by the Denominator (or that Number which is under the Line) and the Quotient shews the Numerator of the Decimal Fraction; As $\frac{3}{4}$ multiply 3 by 100 (by adding 2 Cyphers) makes 300, Divide 300 by 4, and the Quotient will be 75, so that

that $\frac{1}{4}$ in Common Fractions is $\frac{75}{100}$ in Decimal.

C H A P. III.

Of Logarithmetical Arithmetick.

IN the third place the Invention of *Logarithmes* by my Lord Napier, is an excellent Invention; for what is done by Common or Decimal Arithmetick by Multiplication and Division, are done by these artificial Numbers, by *Addition* and *Subtraction* only: The Square Root is extracted by Bipartition, and the Cube Root by Tripartition; to explain these more particularly in all the necessary parts of Arithmetick, as first Numeration in Logarithmes, or how to read them.

S E C T. I.

In the Table of Logarithmes you will find against every Natural Number (from 1 to 1000) its artificial Number or Logarithme; so if you seek the Logarithme of 50, is 1.628970, and if you would find the natural number to this Logarithme 1.62249 over against it you find 42, and so the like

of all others. And what is said of Logarithmes, as to numbers, the like is to be understood all along of Natural Sines and Tangents, and their Artificial Logarithmes, as you may perceive by the Tables.

By the Table of Logarithmes, to find the Logarithme of a Number consisting of 4 places, as suppose 5528, the ordinary Numbers reaching only to 1000.

First, Take the three first Figures, viz. 562, and find that in the first Column under the letter N, then for the last Figure, which is 8, Find amongst the great Figures in the head of the Table, and in the common Area or meeting of these two lines you will find .750354, before which I place 3 for the proper Characteristick makes 3.750354 the Logarithme of 5628, which was required.

Now besides the Logarithme it self, there is also to be placed before it his proper Characteristick, viz. a Figure consisting of an Unite less than the number of digits or places in the said number, whose Logarithme is to be expressed: As the Characteristick of any number consisting of one Figure is 0, of two Figures 1, of three Figures 2, &c. as the Logarithme of 415 is .618048, and because it consists of three Figures, its Characteristick

characteristick is 2 placed thus with a Comma 2,618248.

To find the Logarithme of a proper Fraction, as of $\frac{2}{3}$. First I find the Logarithme of its Denominator 3 to be 0,477121, then I find the Logarithme of its Numerator 2 to be 0,301030: And subtracting the latter from the former, there remains the Logarithme of $\frac{2}{3}$, which is 0,176091.

S E C T. 2.

To Multiply by Logarithmes is to add one Logarithme to another, and that does the same as to multiply the natural Numbers: As to Multiply

24	by	Log. 1.380211
32	is	Log. 1.505150
48		2.885361
72		
768		

Over against which Logarithme you will find in the Table the natural number 768.

S E C T. 3.

To Divide, is to Subtract one Logarithme from another, and that does the same

same as to divide one natural number by another.

As to divide 36 whose Log. 1.556302
by 12 whose Log. 1.079181

is 3

Rems 0.477121

Which Logarithme you will find to answer to the natural Number 3, the Quotient as before.

S E C T. 4.

To perform the *Rule of Three* by Logarithmes. As in *Common Arithmetick* we multiply the Second Number by the Third, and divide by the first (which is the *Direct Rule of Three*) so you must add the Logarithmes of the said Second Number and Third together, and from the Total subtract the Logarithme of the first number, and the remainder is the fourth number sought. As for *Example*.

C 4

If

If 12 give 4, what gives 60.

60	
—	
(0	Log. of 4 is 0.602060
240	of 60 is 1.778151
122	—
x	Added are 2.380211
—	Subst. the third
20	Numbers Log. 1.079181
	—

Rems 1.301030

Which in the Table you will find to be the Logarithme of 20, the Quotient by Common Arithmetick as above.

S E C T. 5.

To Extract the Square Root (which is the same as to find a mean proportional between 1 and the numbers given) therefore to do this observe, That half the Logarithme of the number given is the full Logarithme of the Square Root : So the Log. of 144 is 2.158362, half whereof is the Square Root, viz. the Log. of 12, 1.07918.

S E C T. 6.

To Extract the Cube Root is likewise no more but to take the third part of the Loga-

Logarithme of any Number, whose Cube Root is demanded, which third part is the Logarithme of the same Cube Root: As for *Example*. The Log. of 64 is 1.806180, the third part whereof is 0.602060, which you will find to be the Logarithme of 4, which is the Cube Root of 64; the reason is, because the Cubiq; Root is always the first of two mean proportionals betwixt 1, and the number given:

Note that any Number multiplyed into it self, the Product is the square number, and the Square Root is that same first Number; as 8 multiplyed by 8 gives 64 his Square, and his Root 8. And multiply a Square Number by his Root gives you the Cube; as 64 multiplyed by 8 gives 512 the Cube Number, his Root 8.

S E C T 8.

To find a mean proportional between two Extream Numbers given. Add the Logarithmes of the two extream numbers together; the half of the Sum shall be the Lagarithme of the mean proportional required. As to find a mean proportional betwixt 8 and 32, which shall bear the same proportion to 32, as 8 does to it.

The Logar. of 8 is 0.903090

The Logar. of 32 is 1.505150

Added together is 2.408240

Half whereof is 1.204120

The Log. of 16, the mean proportional,
for as 16 is to 8, so is 32 to 16.

Thus you have Numeration, Multiplication, and Division by Logarithmes: Also the Rule of Three, the Extraction of the Square and Cube Root, and to find a mean proportional between two Extream Numbers given.

He that desires to know more of these, or more at large concerning these, I refer him to the latter end of *Gunters* works, from whence these are for the most part collected.

C H A P. IV.

Of Instrumental Arithmetick.

IN the Fourth place I proceed to that kind of Arithmetick, which I call Instrumental or Lineal, because it is wrought upon Lines and Mathematical Instruments. Those which I shall mention are these Three, Mr.

Gunters

Gunters Sector, Mr. *Wingate's* Rule of Proportion, and Mr. *Seth Partridge's* Sliding Rule, being only the Lines of Numbers, Sines and Tangents diversly improved.

S E C T. I.

The Excellency of *Partridge's* Sliding Rule consists in this, That one may work upon it without Compasses, and after the same be right set, it often resolves several questions at once, without altering the position; only those which I have seen, are set to so short a Radius, which makes it less exact, and more subject to Error. I should advise the Reader to buy the Book, as being very plain, and for the accommodating of it to any other Lines of Numbers, Sines, and Tangents, which may be to a larger Radius, and to use Compasses, observe this general Rule, that what is Termed in the use of that Instrument, [Set such a number on the First to another number on the Second, then against the Third Number on the First, is the Fourth on the Second.] The same upon the plain lines of Numbers, Sines and Tangents, with a pair of Compasses, is thus worded [Extend the Compasses from such a number to such a number] (or from the first number to the second) The same Extent the same way will reach from the Third Number to a Fourth required. This difference

difference observed, the same Book and Directions will serve for either this Rule, or any other line of Numbers, Sines, and Tangents whatsoever.

S E C T 2.

Next as to Mr. *Wingate's* Rule of Proportion, its excellency consists in this, that it is done to a very large Radius, and very neat divisions, and therefore so much the more exact.

Likewise by this you have the Square and Cube Roots by inspection only, and the reason is, because to a very large Line of Numbers, called by him the great Line of Numbers there is adjoined a line of Numbers twice repeated to find the Square Root and mean proportions, called by him the mean line of Numbers, and likewise a line of Numbers 3 times repeated, called his little line of Numbers, and this applied to the aforesaid great line of Numbers finds the Cube Roots, and both these by inspection only.

In other principal Respects both these Rules do agree with the lines of Numbers, Sines, and Tangents upon the outward edge of *Gunters* Sector, he being the first Author and Inventor thereof; therefore having hinted to you wherein the particular Excellencies of these other two Rules do

con-

consist, I shall shew you the general Use of these Lines of Numbers, Sines and Tangents upon the Sector (being the same in substance with the two former, only differently improved) as I said before.

S E C T. 3.

Therefore in the third place, *Gunters* Sector I account the most excellent Instrument of all other, and most general for all Uses, Intents, and Purposes whatsoever; not only in working all proportions, but it may also easily be improved to an excellent Quadrant and Theodolite, thereby to take Heights and Distances, and by that means to become a *Panorganon*, an Universal Instrument to do any thing or every thing both for Calculation and Observation: But in this place I intend only to insist upon what it hath Common with *Partridge's* Sliding Rule, and *Wingate's* Rule of Proportion: To wit,

The Lines of numbers, Sines, and Tangents.

Then to shew you,

I. *Numeration* upon the Lines or the reading of them, which is on this manner. The Line of Numbers consists usually of two parts, the former and the latter, each numbred with these Figures, 1. 2. 3. 4. 5. 6. 7. 8. 9. 1. being the same Line twice repeated.

peated. Now these Figures according as you have occasion, may signifie either single Units, Tens, Hundreds, Thousands, Ten Thousands, or any other imaginable Number, by Mr. *Wingate* called Primes, Tens, Centesmes, Millains, &c. And the smaller divisions signifie proportionably in order to what their Primes are taken for. And in reference to the first and second part of this Line take this Rule. If the Primes that is the large Divisions where the Figures are plac'd do in the former part signifie Units, only the subdivisions are $\frac{1}{10}$ Tenth parts of the same Units, &c. accordingly if smaller Divisions happen under these: And the Primes in the second or latter part of the Line signifie Tens to the former Units, and Hundreds, if the first Primes signifie Tens; Thousands, if the first signifie Hundreds, &c. Now to give one Example for all, the first Prime in the second part is divided first into 10 parts, and those Tens are again really divided into Ten more, and these last at least supposed to be divided into Ten more still; so that suppose a Man would number upon the Line 11575. The first part of the Line signifies the First Figure 10 Thousand. The first Prime in the second part signifies the second Figure 1000. Five of the larger Divisions into Tens signifie 500, the seven subdivisions

divisions into other Tens, signifie 70, and adding to these half of the next small Division by Estimation, signifies the odd 5, so at that very place you have 11575, and so of any other Number, whether bigger or lesser.

For the Line of Sines, 'tis easie being numbered by 10. 20. 30. 40. 50. 60. 70. 80. 90. and the former part with 1. 2. 3. 4. 5. 6. 7. 8. 9. These signifying single Degrees, and the subdivisions first into 6, each signifying 10 Minutes, and those again divided into halves, which signifie 5 Minutes, the rest supplied by Estimation or Supposition: In the second part they are first divided into Ten larger parts, which signifie each one Degree, and those Ten subdivided into 6, which signifie each 10 Minutes, &c.

And what is said of the Line of Sines, the same may be said of the Line of Tangents, signifying also Degrees and Minutes, and proceeding forwards to 45, at the end of the Line and so backwards to 90, where you begun, as you will see signified by Figures to that purpose.

Having as plainly as I can expressed *Numeration* upon the Rule, in the next place I come to shew you the next considerable Rule in *Arithmetick*, upon these Lines: To wit,

H. *Mul-*

II. Multiplication : And to Multiply, Extend the Compasses upon the Line of Numbers from 1 to the Multiplier. This done, the same Extent the same way will exactly reach from the Multiplicand to the Product required : As for *Example*, to multiply 42 by 33, extend your Compasses from 1 to 33, the same Extent will reach from 42 to 1386; so to multiply 18 by 4, Extend the Compasses from 1 to 4 in the former part of the Line, and the same Extent in the second part will reach from 18 to 72. The like is to be understood of all other Questions in Multiplication : Only take this other *Example*.

In Lands and Houses sold at so many years purchase. To find what the value of the whole will be by the Line of Numbers. The Extent of the Compasses from one to the number of years, will reach the same way from the Yearly Rent to the Sum of the Purchase. As the Extent from 1 to 20 will reach from 10 to 200, which shews, that 10 *l. per Annum* at 20 years purchase, is worth 200 *l.* and so of any other.

III. Division : To Divide by the Lines, Extend your Compasses upon the Line of Numbers from the Divisor to One : This done, if you apply the same Extent the same way from the Dividend, the moveable point will fall upon the Quotient. As to Divide

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Divide 12 by 3, Extend the Compasses from 3 to 1, the same Extent will reach the same way from 12 to 4, which is the Quotient.

IV. To Perform the *Rule of Three*, or Three numbers being given to find a Fourth, Extend the Compasses from the First to the Second, and that Extent the same way will reach from the Third number to the Fourth. As to take the same Example as in Logarithmes : If 12 give 4, what gives 60. Extend the Compasses from 12 to 4, the same Extent the same way will reach from 60 to 20, the number required.

V. To Extract the *Square Root* is the same as to find a mean proportionall betwixt 1, and the number given; therefore divide the said distance betwixt 1 and the number given into two equal parts, and at that point you will find the Square Root. As for *Example*, the middle distance betwixt 1 and 9, you will find to be at 3, which is the Square Root of 9, for 3 times 3 is 9.

Now by Mr. Wingate's Rule to find the Square Root by inspection only, take this Rule.

When the Figures of the Number given are even, as when the Number consists of 2 4 6 8 Figures, look the same number in the first part of the mean Line of Numbers, and

and just over against it at the same point upon the great Line of Numbers (or indeed upon that Line of Numbers beginning in the middle, for this together with the other makes but up the great Line of Numbers compleat) there you have the Square Root by inspection only ; as 144 I find at the point S. and over against it 12, which is the Square Root thereof.

VI. To Extract the *Cube Root* is to find 2 mean proportionals betwixt the Number given, and 1 : Therefore upon the Line of Numbers divide the distance betwixt the number given, and 1 into three equal parts, and the first of those 3 parts from 1 is the Cube Root. For *Example*, Extend your Compasses from 1 to 4, and that will divide the distance betwixt 1 and 64 into three equal parts, therefore 4 is the Cube Root of 64, and to prove it, multiply 4 by 4 makes 16, and 16 multiplyed again by 4 makes 64, the Cube number, its Root 4.

Now to do this upon Mr. Wingate's Rule, by Inspection only, Take this Rule.

When the Number propounded consists of these Figures, viz. of 1. 4. or 7. find it in the first part of that which he calls the little Line of Numbers (being the Line of Num-

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Numbers three times repeated.) If it consist of 2, 5, or 8 Figures, then find it in the second part, or if of 3, 6, 9, then find it in the third part : And this done, observe, That at the very same point, upon the great Line of Numbers, you shall find the Cube Root you look for. As for *Example*, To find the Cube Root of 64, because the number consists of 2 Figures, therefore look for 64 in the second part of the little Line of Numbers, and over against it exactly in the great Line of numbers you find 4, which is its Cube Root.

VII. To find a Mean Proportional betwixt two Extream Numbers, is only to divide the space betwixt the Extream numbers into two equal parts, or by *Wingates Rule*, Extend the Compasses upon the mean Line of numbers, from one of the Extream Numbers to the other. This done, the same Extent applied upon the great Line of numbers from either of the numbers towards the other, the moveable point will fall in the middle betwixt them; *viz.* upon the point representing the mean proportional required. *Example*, 8, 16, 32.

Thus you have all the necessary parts of Arithmetick performed by the Lines of Numbers, Signs, and Tangents, and what is said as to Numbers only, the like will hold as to Signs and Tangents alone, or these several

verals mixt, as between Sines and Numbers, Tangents and Numbers, or Sines and Tangents; as the nature of the question shall require. He that desires to know more concerning these, I should advise to buy Mr. *Wingate's* Book, called his Rule of Proportion, wherein you have the Rule it self in Paper, which you may get pasted upon wood; and will be no great cost, and will shew you the whole Mystery of it; also I should advise to buy *Seth Partridge's* Book, shewing the use of his Sliding Rule, being plainer than *Wingate's*, only observing the Rule before given for applying the Questions, so as to be wrought upon a plain Line of Numbers, Sines, and Tangents, and a pair of Compasses.

S E C T. 4.

To conclude what may be said in reference to *Instrumental Arithmetick*, I think it not amiss to give you a short Account of *Napiers Bones*, whereby Multiplication and Division in *Common Arithmetick* is performed by Addition and Subtraction only without charging the Memory.

Napiers Bones then are nothing else but *Pythagoras Table* with divided Lines. For Reading the Bones begin always on the right hand, and take them as they lye in each Diagonal Line, and if there be two Figures

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figures in the same Diagonal, make but one of them by adding them together; and if they exceed 10, take 1 to the Figure of the next Diagonal immediately following. As for *Example*.

Against 9 you find in the first Diagonal 2 by it self; in the next 7 and 4 which makes 11; therefore set down 1, and carry one to the next Diagonal, which is 5 and 1, and 1 you carryed makes 7, and in the last Diagonal stands 8 by it self, so they are to be read 8712, and so of any other number whatsoever,

0	6	8	1	Multiplicand	9	6	8
1	1	1	2	Multipliyer	5	7	9
2	2	2	3				
3	3	3	4				
4	4	4	5				
5	5	5	6				
6	6	6	7				
7	7	7	8				
8	8	8	9				

8	7	1	2
6	7	7	6
4	8	4	0
5	6	0	4
7	2		

Product

In

In Multiplication place always the Multiplicand in the Top of the Bones, and against every Unite of the Multiplier in the side, you will find the particular Product of the same Unite, which added together, as in the *Example*, makes up the whole Product as before.

For *Division* place the Divisor in the top of the Bones, and by finding the nearest less number to the Dividend, you may find how often the Divisor is contained therein.

As for Example.

Divisor 968
Dividend 6776

Say how many times 968 in 6776. Look 6776 (or the nearest less number to it) but this is in the Table over-against 7, which shews it is just 7 times contained in the Dividend; so that 7 is the Quotient without any Fraction, and the like is to be understood of dividing any other number by any other number: Or to give you an Example how to divide a larger Sum, and by consequence any Sum; as to Divide 560472 by 968, by *Napiers Bones*.

First, Upon Paper set down the Dividend, and making a stroke with your Pen on the left hand, there place the Divisor, and

and making another stroke with your Pen on the right hand, there place the Figures of the Quotient, then observe upon the Dividend how far the Divisor will reach, as in this Example to the Figure 4; there make a prick; then having tabulated the Bones, placing the Divisor in the Top as before, find out the nearest less number to 5604, which you will find to be 4840, which you will find against the Figure 5, therefore put 5 in the Quotient, and place this 4840 under the Figures of the Dividend, and subtract it from 5604, there remains 764, to which add the next Figure in the Dividend, *viz.* 7. (making a prick under the said Figure in the Dividend, to know how far you are gone in the operation) These together make 7647: Then upon the Bones look for the nearest less number to 7647, and you will find it 6776, and over against it the Figure 7, therefore put 7 in the Quotient: And Subtract this 6776 from 7647, the Remainder is 871, to which add the next Figure in the Dividend, *viz.* 2, and these make together 8712: Therefore on the Bones look the nearest less number to 8712, but you will find the exact number it self standing against the Figure 9, therefore put 9 in the Quotient, and nothing remains, else what had remained had been the Fraction, and this will

will make plain any other operation in Division upon *Napiers Bones*, and therefore I shall need to add no more.

$$968)560472(579$$

$$\begin{array}{r} 4840 \\ \hline \end{array}$$

$$\begin{array}{r} 7647 \\ \hline \end{array}$$

$$\begin{array}{r} 6776 \\ \hline \end{array}$$

$$8712$$

And thus much shall suffice to have spoken for the understanding of *Napiers Bones*. As also for what I shall say to the First Part of the *Mathematicks*, consisting in *Arithmetick*, and the four several kinds of it, as it is distinguished into *Common*, *Decimal*, *Logarithmetical*, and *Instrumental*; and here I intended to have shewn the Use of the Rest of the *Sector* (besides the Lines of Numbers, Sines and Tangents :) But upon more mature consideration I find it more properly to be *Geometrical* than *Arithmetical*, and therefore by way of Transition it will lead me by the hand (as it were) to the Second Part of the *Mathematicks*, (or the Second *Mathematical Liberal Science*) to wit, *Geometry*, where you may expect an Explanation of the rest of the *Sector*, as also the way of *Protraction* by Scale and Compass. Of

The Second BOOK.

OF

GEOMETRY.

PART I.

G EOMETRY is the Second *Liberal Science* which may be properly called *Mathematical*, and may be considered under a double Notion. First in General, as it makes and measures Geometrical Figures. Secondly according to the proper and particular Etymology of the Word, which is derived from *γῆ Terra*, & *μετρέω Mensura*, The Measuring of the Earth (that is as I have elsewhere explained it) the whole Globe of the Earth, containing Sea and Land, that which concerns the Land being called *Surveying*, and that which concerns the Sea, *Navigation*.

D

I. But

I. But first of Geometry in General.

All Bodies whatsoever which come within the compals of Geometry, or are to be measured may be distinguished into these Two kinds. First, *Superficies*. Secondly, *Solids*. And these are represented by *Geometrical Figures*, which Figures being protracted or laid down in due Proportion and Quantities answerable to the several Bodies they represent. The Bodies themselves are Measured by the same Rules as these Figures are which represent the same.

Now these Figures are as various, as there are Superficial or Solid Bodies of various and different shapes and magnitudes; inso-much as in this respect they may be said to be infinite, some Regular, but the most Irregular. All which, notwithstanding of what shape or irregularity soever, may be reduced to some of these three Regular *Geometrical Figures*, the *Circle*, *Square*, and *Triangle*. Therefore in this place I shall shew you

1. How these three Figures are produced, protracted, or made.
2. How they may be reduced to one another, or any other to these, or some of these. And,

3. How

3. How these, and consequently all other Geometrical Bodies or Figures are to be measured, whereby you will see, that as *Arithmetick* is principally comprized in the *Rule of Three*, so *Geometry* and *Astronomy* may be comprehended in the Resolution, or the making and measuring of a Triangle, called *Trigonometry*.

First then, How these three Figures, the Circle, the Square, and the Triangle are produced, and so they will be found to derive their Original from these three Geometrical Principles.

1. A Point which is void of all magnitude, and hath neither Length nor Breadth.

2. A Line which is a Length without Breadth, drawn from one point to another, and may be either strait or crooked.

3. An Angle which is made by two Lines meeting in a point, that point being called the Angle, or the Angular Point: So that as the *Terms* or *Limits* of a Line are Points, and the *Terms* or *Limits* of Angles are Lines, so the *Terms* or *Limits* of Superficies are both Lines and Angles, and the *Terms* or *Limits* of Solids are Superficies.

Again, a Point hath neither Length nor Breadth; a Line hath Length, but no Breadth; a Superficies hath both Length and Breadth; a Solid hath both Length, Breadth, and Thickness.

C H A P. I.

BUT before I speak particularly of these three *Geometrical Figures*, I should necessarily lay down some few *Geometrical Problems*, because (as a Circle is made of a Line drawn equally distant on all sides from a Point called its Center.) So the Square and the Triangle are made up of Lines and Angles, and therefore there will be several times mention made of Parallel Lines, and Perpendiculars: And also in the two latter Figures there will be occasion for laying down of Angles, &c. I shall therefore in this place (once for all) lay down these few *Geometrical Problems*, not to trouble you with any more but those absolutely necessary.

S E C T.

S E C T. I.

First, To draw a Line Parallel to any other Line given at any distance required.

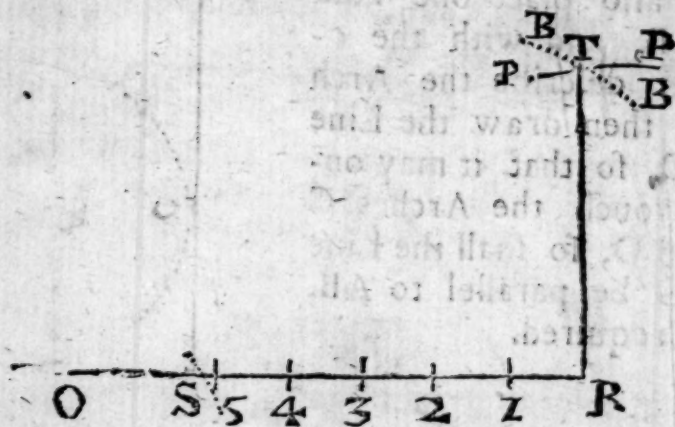
Let the Line given be AB, unto which it is required to draw another Line Parallel thereunto at the distance AC, or BD. Open your Compasses to the distance AC or BD, and place one Foot in A with the other describe the Arch C, also place one Foot in B, and with the other describe the Arch D, then draw the Line CD, so that it may only touch the Arches C and D, so shall the Line CD be parallel to AB, as required.



S E C T. 2.

To Erect a Perpendicular on the end of a Right Line given.

First, Divide the Line OR from R in any 5 Equal parts, as in the Example 1, 2, 3, 4, 5. Then take with your Compasses the distance from R to 4, and placing one Foot in R, with the other describe the pricked Arch PP, then take with your Compasses the distance from R to 5, and placing one Foot in 3, with the other describe the pricked Arch BB, and from the point where these two Arches intersect one another to the point R, draw a Right

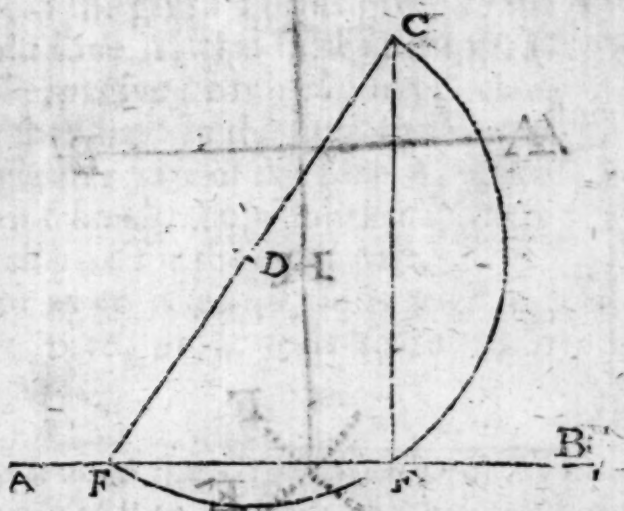


Line, and that will be a true Perpendicular: or open your Compasses to any convenient Distance, and set one Foot in the point

point R, with the other make a mark at pleasure, as $\odot$, then keeping the Compass point in $\odot$, with the other describe an Arch as BB, as also another Arch to cut the given Line in S. Lastly lay a Ruler from S to $\odot$, will cross the Arch BB in T, and a line drawn from I to R will be the perpendicular required.

Proposition 3. How to let fall a Perpendicular from any point assign'd upon a Right Line.

Let the Point given be C, and to let fall a Perpendicular upon the Line AB. First from C draw a Line by chance, as CE, which divide into two equal parts at D,



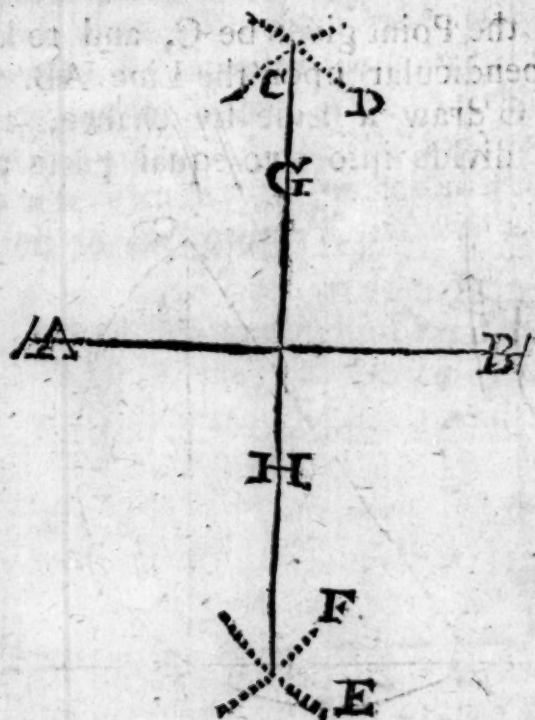
and with the distance DC describe the Semicircle CFE, and note where the same

cuts the Line AB, and from that point A, to the point C, draw the right Line CF, which shall be the Perpendicular required.

S E C T. 4.

To Divide a Line into 2 equal parts by a Perpendicular.

Let the given Line be AB, fix one foot of your Compasses in A, and setting them at any convenient distance above half the given line, describe the Arches D and F, carry your Compasses to B with the same



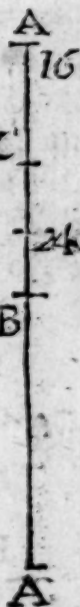
distance describe the Arches C and E, so as they may cross the other Arches, and by those Intersections lay your Ruler, and draw the line GH, which will cut the given Line in the midst, and be a perpendicular also to it, which was required.

S E C T. 5.

To Divide a Line given to any Proportion required, as to Divide the Line A containing 40 Equal parts (let them be what they will) in proportion, as 20 is to 30, or as the Line B is to C.

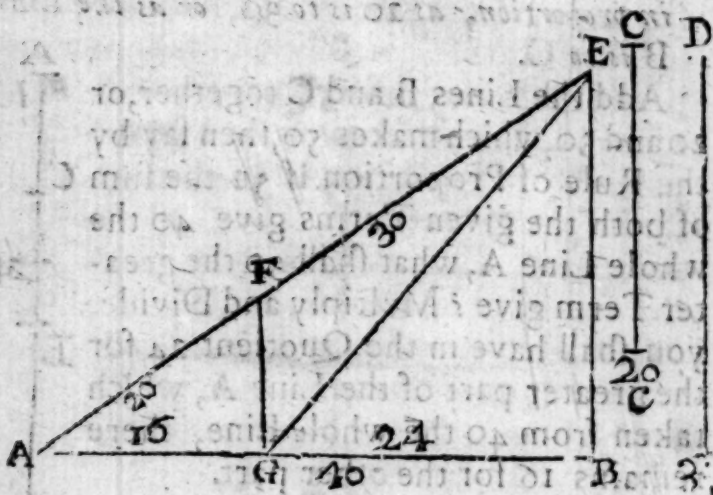
Add the Lines B and C together, or 20 and 30, which makes 50, then say by the Rule of Proportion. If 50 the sum of both the given Terms give 40 the whole Line A, what shall 30 the greater Term give? Multiply and Divide, you shall have in the Quotient 24 for the greater part of the Line A, which taken from 40 the whole Line, there remains 16 for the other part.

For as 50 is to 40, so is 30 to 24, or 20 to 16, or as 30 is to 20, so is 24 to 16.



But to do this *Geometrically*, or by Lines, suppose AB to contain 40, and you would divide the same in such proportion as C to D.

First, From the point A, draw the Line AE at pleasure, making the Angle EAB, then take in your Compasses the Line C, and set it from A to F, also take the Line D, and set it from F to E, and draw the Line EB perpendicular to AB, Then from the point F draw the Line FG parallel to EB, cutting the given Line AB in G, so is the Line AB divided as required.

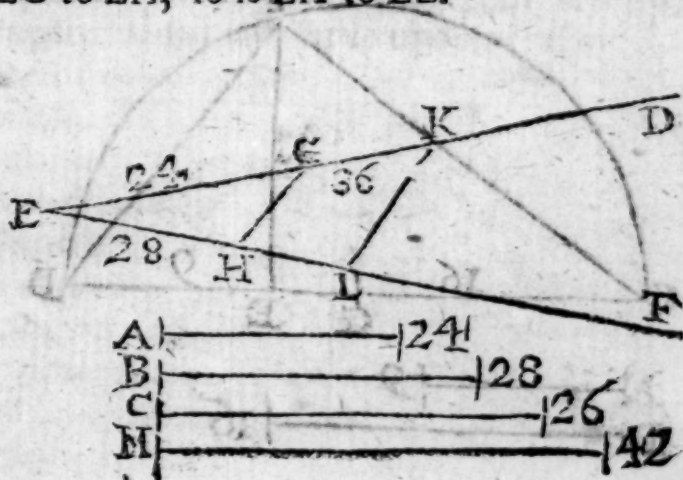


By dividing the Base of a Right Angled Triangle by this Problem, and drawing a Line from the opposite Angle to the point of Division at G, you divide the Triangle it self according to the same proportion.

S E C T. 6.

Three Lines being given to find a fourth in proportion to them. Or to perform the Rule of Three in Lines, Draw 2 Lines, making any Angle as the Lines ED and EF, then the Three Lines given, being ABC, to find the Line M.

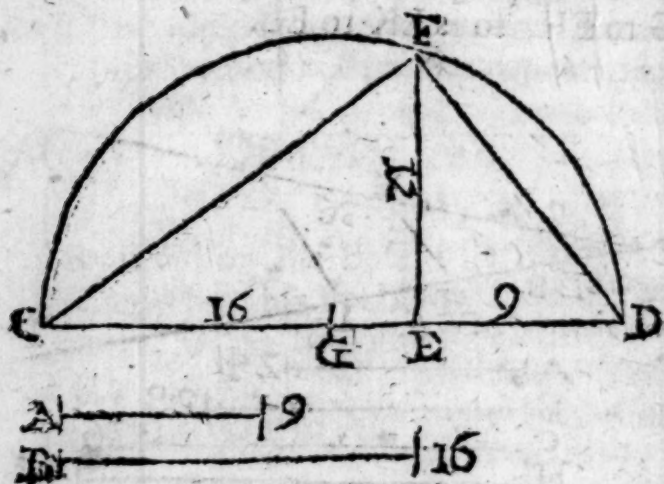
First take the Line A in your Compasses (that is 24 from a Scale of equal parts) and set it from E to G, then take the Line B 28, and set that length from E to H, then take the third given Line in your Compasses C 36, and set it from E to K, and through the point K draw the Line KL, parallel to GH, so shall the Line EL be the fourth proportional required. For, as EG to EH, so is EK to EL.



S E C T. 7.

To find a mean proportional Line between Two Lines given.

Let the two Lines given be A and B, which join together in the point E, making one right Line as CD which divide into two equal parts in the point G, upon which point G with the distance GC or GD describe the Semicircle CFD, then from the point E, (where the two Lines are joyned together) raise the perpendicular EF cutting the Periphery of the Semicircle in F, so shall the Line EF be a mean proportional between the two given Lines, A and B. For, As ED. 9. is to EF. 12. so is EF. 12. to CE. 16.



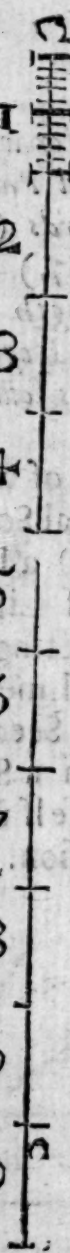
S E C T.

S E C T. 8.

To Protract by Scale and Compass, or for want of a Scale to make a Line of equal Parts, which together with a Line of Chords (the making whereof is taught Folio 67.) You may lay down any Line of what length you please : or any Angle of how many Degrees and Minutes soever, which take as follows.

First, For laying down any Line of what length you please. There are several Scales both of Brass and Wood, (that is) a Line of equal parts for protracting of Lines; and a Line of Chords for protracting of Angles ; (together with other Lines at pleasure, as a Line of Rhumbs and Secants, or the like :) But for want of such a Scale you may make these Lines your self upon Paper, which will serve upon occasion.

First



First then, To make a Line of equal Parts, open your Compasses to some small distance at pleasure, and run it ten times over. Then opening the Compasses to the Extent of the whole Ten small Divisions run that distance over nine times more, to have you a Scale of equal parts, and one of them divided into other ten parts. Then taking 6 of these larger Divisions in your Compasses, make it the Radius or Semidiameter of a Circle, whereby you may draw a Line of Chords, or Rhumbs, or Secants, &c. as you'll see hereafter.

Now to Protract by these you shall be taught in making those 2 following Geometrical Figures, viz. Squares and Triangles. To avoid unnecessary Repetitions, there are also *Protractors* (properly so called) by which you may lay down angles, and avoid the drawing of many unnecessary Lines on the Paper; which said *Protractors* are made of a thin piece of Brass, cut into a perfect Semicircle, and divided into Degrees of a Circle, with a hole in the Center, and a Diameter drawn through the said Center, which is called the *Meridian Line*. The way of Protracting by this Pro-

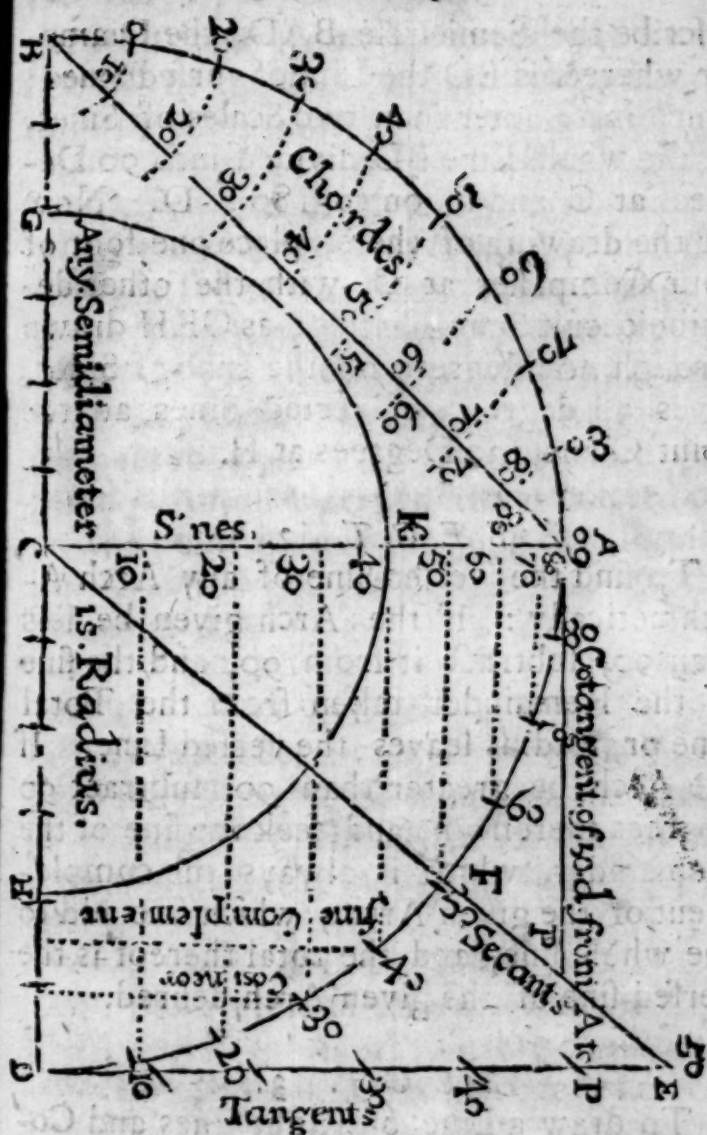
Protractor, you shall also have exemplified in making of Squares and Triangles. And in the third place you may also Protract Angles from a Line of Right Sines as shall also be shewed you hereafter; and shall give you examples of each of these several ways in every Figure, so you may make use of which pleases you best. But now to proceed to speak of those three famous Geometrical Figures, The Circle, Square, and Triangle particularly, and in order, and that in a double Capacity. First, as they are Superficies, And, Secondly, as they are Solids. And first of a *Circle*.

C H A P. II.

Of a Circle.

A *Circle* is a Geometrical Figure, which of all other is the most perfect, being an Emblem of Eternity, having neither Beginning nor End, and is made by the drawing a Line called the Circumference, equally every way distant from a Point in the middle called the Center, thorough which Point or Center a straight line drawn from any part of the Circumference to another is called the Diameter; and any Line
drawn

drawn from the said Center to the Circumference is called the Semidiameter or Radius, or *Sinus Totus*, the whole Line of Sines, which being divided into 90 unequal parts are called Right Sines. The whole Circumference of the Circle, let it be bigger or less, is always supposed to contain 360 Degrees, and is accordingly by Mathematicians always divided into so many equal parts. The Semicircle into 180, and every Quadrant or fourth part of a Circle into 90. Each of these Degrees are subdivided into 60 Minutes, and each Minute supposed at least to contain 60 Seconds, and every Second 60 Thirds, &c. as the largeness of the Circle will admit. Now by means of these Divisions in the Circumference, for the finding out of Proportions, besides the Line of Sines before-mentioned, there are many other Lines belonging to a Circle; as there are not only Right Sines, but also Versed Sines, Sines, and Sines complements, or Cosines; Tangents, and Cotangents, Secants, and Chords, for the understanding of all which I shall give you this following Diagram, and how all these Lines are drawn and divided.



S E C T. I.

First then, Setting one Foot of your Com-
passes in the Center at C , with the other
describe

describe the Semicircle BAD, the Diameter whereof is BD, the Line of versed Sines; which is no other than two Scales of Sines, as the whole Line BD divided into 90 Degrees at C and so on to 180 at D. Now for the drawing of these, place one foot of your Compasses at C, with the other describe occult Semi-Circles; as GKH drawn through 40 Degrees of the Line of Sines, gives 50 degrees of versed Sines at the point G and 130 Degrees at H.

S E C T. 2.

To find the versed Sine of any Arch Arithmetically: If the Arch given be less than 90, subtract it from 90, and the sine of the Remainder taken from the Total Sine or Radius leaves the versed Line. If the Arch be greater than 90, subtract 90 degrees therefrom, and seek the sine of the Remainder, which is always the complement of the given Arch; which sine add to the whole sine, and the total thereof is the versed sine of the given Arch desired.

S E C T. 3.

To draw a Line of Right Sines and Cosines through every Degree, or Tenth Degree of the Quadrant AD, draw right Lines parallel to the Line CD, till they cut the Lines AC, and those are sines. A Line drawn
per-

perpendicular from the same place to the line CD is the sine complement or Cosine, as in the Diagram.

S E C T. 4.

To draw a Line of Tangents and Cotangents. For Tangents: upon the point D erect a perpendicular DE, and through every degree of the Quadrant AD draw right Lines from the Center C, till they cut the perpendicular DE, and these are Tangents. Another perpendicular erected upon the point A, and touching the aforesaid line drawn from the Center, are Cotangents, as AP.

S E C T. 5.

To draw a Secant is this very line drawn from the Center to E, which is the Secant of 50 degrees, the perpendicular it self, which it cutteth, being the line of Tangents.

S E C T. 6.

To draw a line of Chords, Draw a strait line from B to A, and setting one Foot of your Compasses in B through every Degree, or every tenth Degree in BA, draw Arches to cut the straight line, and that shall be a line of Chords.

All these lines are of admirable use in finding of proportions, arising from that Symmetrical

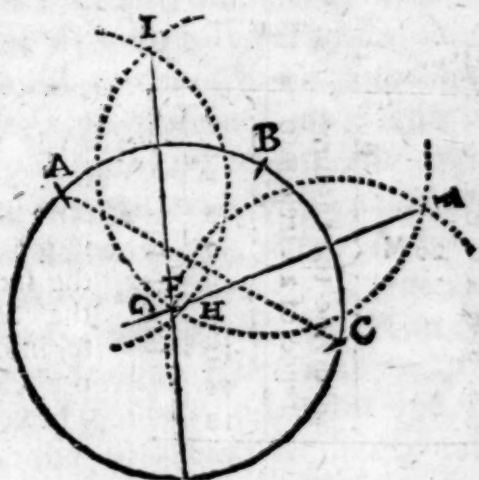
metrical proportion there is betwixt a Circle, and the parts and lines belonging thereunto ; for according to the length of the Radius or Semidiameter, so is the Circumference ; and so are the Sines, Tangents, Secants and Chords belonging to the same Circle or Radius : For Sines and Tangents you see what excellent Effects they have joyned with the line of Numbers in *Instrumental Arithmetick*. And for the Line of Secants, it is that which constitutes the Meridian Line, or Line of Latitudes of absolute use in Navigation ; and for the Line of Chords, its excellency consists in protracting of Angles, without which (excepting the Circle, which is perfectly round,) no other Geometrical Figure can be made or measured.

P R O B L. I.

Three Points being given, not lying in a direct Line, to find the Center of a Circle, which shall cut all the 3 Points.

Let the Points be A B C, open your Compasses to some reasonable Scantling more than half the distance betwixt A and B, with that wideness strike a Portion of an Arch upon the point A, do so also upon the point B, and note the intersections of these

these 2 Arches, as at I and H, and produce a line from I by H infinitely. Then with your Compasses at the same distance as be-



fore, setting one foot in B ; strike an Arch towards C, and removing the same to C, strike another Arch towards B, to cross each the other, and where these intersect as at F and G, draw another strait line to cross the former ; and where these two strait lines intersect each other, as at E, is the Center of a Circle, which shall cut all the 3 points ABC the thing required. The same Problem will also find the Center to any Segment of a Circle.

To Divide the Circumference of a Circle into any Number of equal parts. Divide 360 by the Number of Parts, the Quotient shews the Degrees as follows.

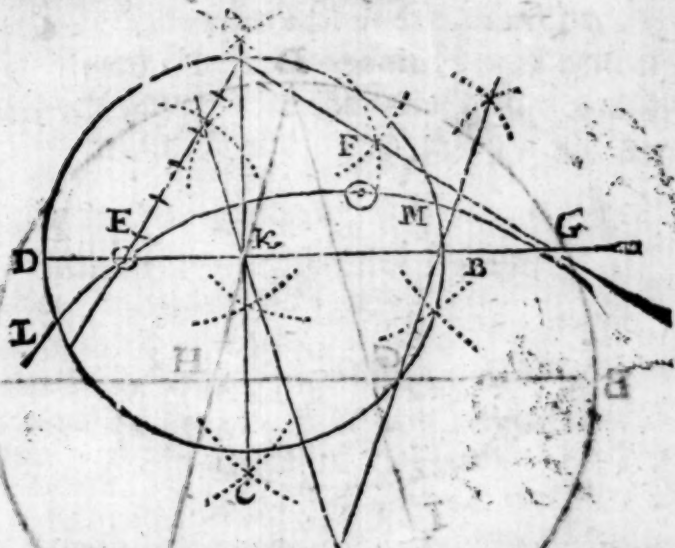
A Chord of	120	will divide the Circumference of a Circle into	3	Equal Parts.
	90		4	
	72		5	
	60		6	
	51 26 M.		7	
	45		8	
	40		9	
	36		10	
	32 44		11	
	30		12	

And note that the Semidiameter will divide it into 6 parts, and the sixth part of the Semidiameter into 36 equal parts, &c.

P R O B L E M 2.

Two Points within any Circle being given, how to describe the Arch of another great Circle which shall pass through those two Points, and also divide the Circumference of the given Circle into two equal Parts.

Let the two Points be E F, through either of which (as E) draw the right line DE, so as to pass through the Center K: at right Angles draw the line AC, then draw the line EA, and from the point A erect a perpendicular AG, and so have you three points EFG, through which 3 points describe

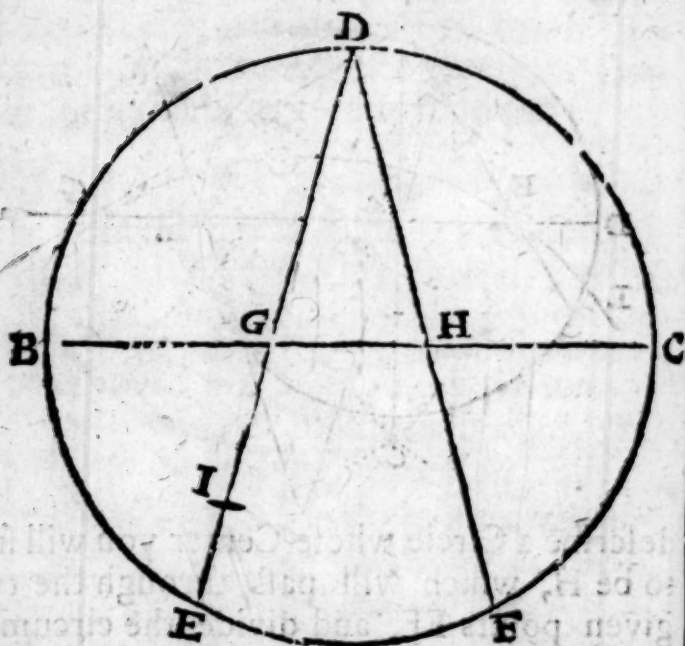


describe a Circle whose Center you will find to be H, which will pass through the two given points E and F , and divide the circumference of the given Circle into 2 equal parts at L and M .

P R O B L. 3.

To find a Right Line equal to the circumference of a Circle given.

Let the given Circle be $BDCFE$, Divide the upper Semicircle into halves at D , and the lower Semicircle into three equal parts at F and E , and draw the lines DE , DF , which cut the Diameter of the Circle at G and H . Then make GI equal to GH . And the length DI is a very little more than the length of the quadrant BD , neither



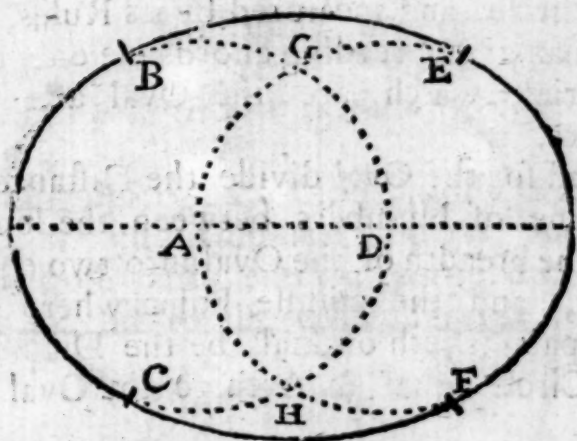
ther doth the excess amount unto one part of the Diameter BC, if it were divided into 5000, and four times the extent DE will be a little more than the whole circumference of the Circle.

P R O B L. 4.

To draw a Geometrical Oval.

First, Draw two equal Circles, whose Centers are A and D, the Distance AD being equal to the Radius or Semidiameter of each Circle : Then open your

Com



Compasses to the whole Diameter of the Circle, and set one Foot in H, with the other strike the Arch B G E, and with the Foot of the Compasses removed to G strike the Arch E H C, so will the Figure BCFE be a perfect Geometrical Oval.

C H A P. III.

HAVING shewn you how a Circle is produced, with all the Parts and Lines belonging thereunto; I come to shew you in the next place, how other Figures may be reduced thereunto; and afterwards shall shew you how to measure a Circle either Superficial, or Solid.

E

Now

Now the Figures which may be reduced to a Circle, and measured by its Rules, are not many: My reading affords me only two material, which are the Oval and the Square.

And for the Oval divide the Distance on the line of Numbers between the length and the breadth of the Oval into two equal Parts, and the middle Point where the Compass stayeth on shall be the Diameter of a Circle equal in Area to the Oval given.

SECT. I.

To Reduce a Circle into a Geometrical Square, or by knowing the Superficial content of a Circle, to find the side of a Square equal to it.

Extract the Square Root of the whole content of the Circle, by taking half the Logarithme of the said content. As suppose the content to be 616, the Logarithme whereof is 2,789581.

Take the half thereof, } which is the Square Root } 1.394790 $\frac{1}{2}$

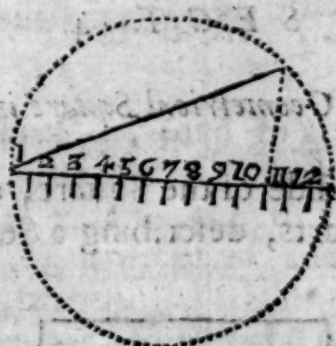
The nearest number answering hereto is 25, so the side of the Square ought to be near 25 of the said parts, whereof 616 were supposed to be the contents, whether Inches, Feet, Yards, &c.

SECT.

S E C T. 2.

To Reduce a Circle into a Geometrical Square.

Having described a Circle, draw a Diameter, which divide into 14 parts, and on 11 of those parts erect a perpendicular, which extend to the circumference, and



from the point of intersection, draw a line to the extreame part of the Diameter, which Line shall be the side of the Square, containing the proportion (or Area) of the Circle.

S E C T. 3.

To Reduce a Square to a Circle Instrumentally.

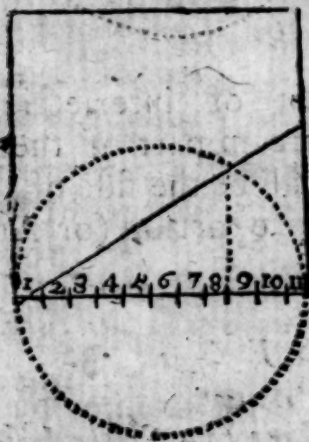
The side of the Square (as above) is the Square Root of a Number, which is the Superficial

perificial content of a Circle equal to the said Square, and knowing the superficial content, you may find the Diameter. If the Diameter be 10, the Superficial content upon the line of Numbers is 78.54 parts. Divide the distance betwixt the given content, and 78.54 into two equal parts. The same distance the same way will reach from 10 to the Diameter required.

S E C T. 4.

To Reduce a Geometrical Square into a Circle.

Take any side of the Square, and Divide it into 11 parts, describing a Semicircle to



the said Diameter: Then on 8 of those parts erect a perpendicular, which extend

to

to the circumference of the Semicircle. Then draw a Line from the extream part of the Diameter, intersecting the perpendicular in the circumference of the Circle, and continue that line till it touch the side of the Square; so that line will be the Diameter of a Circle answerable to the Square given.

C H A P. IV.

TO conclude, (and to get out of this Circle) it remains only to shew you how to measure a Circle, and consequently an Oval, a Square, or any other Figure reducible thereunto.

First then for a Superficial or Flat Circle, such as may be a piece of Ground, or Board, or Glass perfectly round, or any other Flat, Round Superficies. Take these Rules.

S E C T. I.

Having the Diameter to find the Circumference.

Multiply the Diameter by 22, and Divide the Product by 7, as 28 by 22 is 616. Divided by 7 is 88.

S E C T. 2.

Having the Circumference, to find the Diameter.

Multiply by 7, and divide by 22. As 88 multiplied by 7, gives 616 divided by 22, is 28 the Diameter.

S E C T. 3.

By the Diameter only to find the Area, or Superficial Content.

Multiply the Diameter by it self, as 28 by 28, that makes 784. This Product multiplied the second time by 11 is 8624, which divide by 14, the Quotient will be 616, and that is the Area of the Circle.

For a half Circle, a Quadrant or any lesser portion of a Circle, whose Point goeth to the Center. Half the Arch multiplied by the Semidiameter, produceth the Superficial Content.

To measure the content of the Segment of a Circle, Take the Chord $12\frac{2}{3}$, and the Perpendicular 4, and multiply the whole of the one by two Thirds of the other, and it will come very near.

To find the Superficial content of a Circle, you need do no more but multiply half the circumference by half the Diameter. As the Circumference being 44, the
half

half is 22, the Diameter 14, the half is 7. Multiply 22 by 7, and the exact content of the Circle is 154.

To find the whole Diameter of a Circle, by knowing part thereof, and the length of the Chord crossing the Diameter in that part. Suppose part of the Diameter to be 4, the Chord intersecting it $12\frac{1}{3}$, Square one half of the Chord $6\frac{1}{3}$, multiplied by it self is 40, divided by 4 the part of the Diameter given, rests 10, which added to the said part, shews the whole Diameter to be 14, whether the Section be of a Circle, or of a Globe.

C H A P. V.

Of a Solid Circle, such as are Globes, Spheres, Bullets, Balls, or the like : To find their Solid Contents.

Multiply the diameter of the Circle by it self, as 28 by 28 makes 784, this Product multiplied the second time by the Diameter 28 makes 21952, which two Multiplications are called cubing of the Diameter. Then multiply that Cube Number by 11, makes 241472, and divide this

last product by 21, produceth the Solid Content 11498 $\frac{4}{5}$.

S E C T. I.

To find the solid Content of a Globe another way.

Find first the Superficial content of a Globe, which to do, if you multiply half the diameter by half the circumference, the Product is the Superficial content of a Circle of equal diameter. As 28 multiplied by 88 is 2464 : And then if you Multiply the content of a Circle having the like diameter by 4, the Product is the Superficial content of a Globe, as 616 multiplied by 4, produceth likewise 2464.

Now hereby to find the solid Content of a Globe.

Multiply the Superficial content by the sixth part of the Diameter, the Product shall be the solid content of the Sphere.

S E C T.

S E C T. 2.

In like manner knowing the solid content of a Globe, to find the side of a Cube equal to the Globe.

Extract the Cube Root of the solid content of the Globe, which is done by taking the third part of the Logarithme of the said solid content. And this Cube Root shall be the side of a Cube, which in solid contents shall be equal to the Globe given.

S E C T. 3.

And to find the Superficial Content of the Segment of a Globe; say, If the whole Diameter 14 give the whole Superficial Content 616, what will 4 the part of the Diameter give, which you will find 176, and so of any other.

And thus much shall suffice to have spoken of a Circle, how it is produced, reduced, and measured.

Having hitherto treated of a Circle and Globe, I come now to shew you the Producing, Reducing, and Measuring of the other two principal Geometrical Figures, the Square and Triangle, as I have already done

concerning the Circle. And as the Circle was made up of a point, and a crooked or circular line drawn round about it ; so Squares and Triangles are made up of Lines and Angles ; Squares, and right lined Triangles of streight Lines and Angles, Spherical Triangles of crooked circular Lines and Angles : Now a streight, or a right line is the nearest distance drawn betwixt two points.

A circular or sphærical Line is part of the circumference of a Circle bigger or less.

An Angle in a Spherical Triangle, is (as was said before) the distance of two Lines meeting in the Angular point, and measured in the circumference of a Circle at the distance of the Radius, or Semidiameter of the said Circle from the said Angular point, which pray observe very well.

Now every Circle, whether great or little, doth contain 360 degrees, every degree containing 60 Minutes, so that a Semicircle contains 180 degrees, a Quadrant or fourth part of a Circle 90 degrees. Now if an Angle be a Quadrant of a Circle, and contains 90 degrees, it is called a Right Angle : If it contains less than 90 degrees, it is called an Acute Angle, if more than 90 degrees, it is called an Obtuse Angle. These Angles and Lines do constitute all kinds of Geometrical Figures whatsoever.

C. H. A. P.

CHAP. VI.

Of a Square.

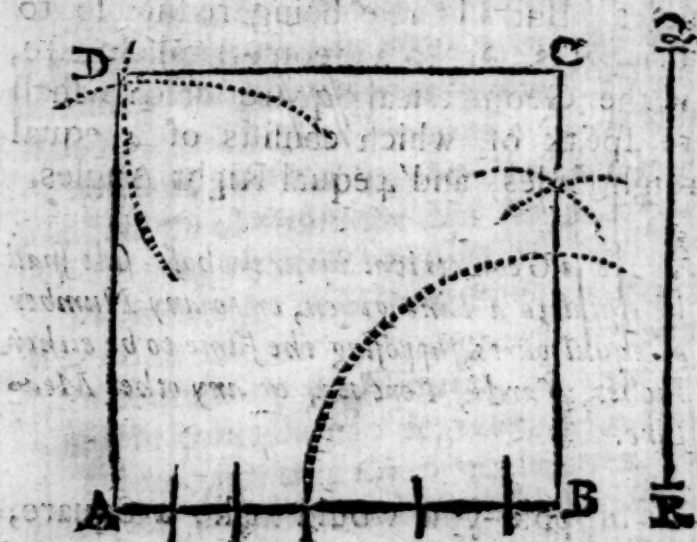
THere are several sorts of four sided Figures, as the Long Square, or Parallelogram, whose Angles are all equal, a Rhombus, and a Rhomboides, whose opposite sides are all equal; and a Trapezium consisting of 4 unequal sides, and 4 unequal Angles: But all these being reducible to 2 Triangles, or to a Geometrical Square, it is the Geometrical Square which I shall here speak of, which consists of 4 equal Streight Sides, and 4 equal Right Angles.

To make a Geometrical Square whose side shall be equal to a Line given, or to any Number of equal parts, supposing the same to be either Inches, Yards, Perches, or any other Measure.

As suppose you would make a Square; whose side should contain 36 equal parts, (which suppose yards) or whose side should be equal to the line QR, which is the same; First, From your line of equal parts, either upon your Scale, or prickt out upon Paper,

take

take with your Compasses three of the larger divisions, which signifie 30, then extend them further to 6 more of the smaller divisions, which together make 36, and prick out, and draw a line of the same length as AB : then by the *Probl. fol. 54*, erect a perpendicular upon the end of the same line at B, containing the same length. Then keeping your Compasses at the distance of QR, or AB, or BC, (all three being equal) set one Foot of your Compasses in the point C, with the other



describe the Arch at D: Also the compasses resting at the same distance, place one foot in A, and with the other cross the former prickd Arch in the point D. Lastly, where

where these Arches intersect one another, make a prick with the point of your compasses, and from thence draw right lines from D to C, and from D to A, which shall include the Geometrical Square, ABCD.

Now if instead of erecting the perpendicular BC(as is before directed)you protract an Angle of 90 degrees, or a right Angle at the point B, it will effect the same thing, and a line drawn at the same Angle will be a true perpendicular.

Now I told you before, there were 3 several ways of protracting an Angle.

1. By Scale and Compass, or a line of Chords.

2. By a Protractor properly so-called.

3. By a line of Right Sines, and I shall shew how to protract an Angle of 90 degrees at the point B, every one of these ways.

S E C T. I.

First, By a line of Chords; and note that you must either have a line of Chords, whose Radius is shorter than the line AB, or DC, or else you must produce these lines till they meet with the Radius. Then with your Compasses set one foot at the beginning of your line of Chords, and the other extend to

to 60 (where usually are brass Centers in most Scales) and with that extent setting one Foot in B, describe an Arch of a Circle, cutting the lines AB and CB, and then from the points where this Arch intersects the lines AB CB, measure with your compasses in the same Arch the Chord of 90 degrees, which is the distance from the beginning of the line of chords to 90 degrees or the whole line, and making a prick there, lay your Ruler to the said prick and the point B, drawing a line to the point B, and the Angle at B contained betwixt the two lines AB and BC shall be a Right Angle, or an Angle of 90 degrees, which was the thing required; and the like is to be done for any other Angle, of what quantity of degrees. Soever, 15, 30, 45, 50, 130. degrees, with their odd Minutes, if there be any; but I hope this Example may serve for all, or instead of many, so I shall mention no other in this place.

S E C T. 2.

Secondly, By the Protractor do thus, put a Pin through the Center hole, fixing it in the point B, then lay the Meridian Line of the Protractor upon the Line AB, and by the edge of the Protractor make a mark against 90 degrees, and from that mark draw

draw a line to the point B, and the Angle included betwixt the two lines is an Angle of 90 degrees, as before. Suppose the Semi-circle BAD, (as in pag. 65) were a Protractor, the Limb thereof divided into degrees as there it is, the point C is the hole at the Center, the Line BD is the Meridian line, and at the point A you find 90 degrees: The like is to be understood of any other Number of degrees whatsoever, more or less, or any other Protractor whatsoever. Lay the Meridian upon AB the Line given, the Center upon B, and against 90 degrees, or the point A in the Protractor make a mark at C, and from C to B draw a Line, and AB, and CB will include an Angle of 90 degrees as required.

PROPOSITION 3. To draw a Line perpendicular to a given Line.

S E C T. 3.

Thirdly, To Protract an Angle (which suppose 90 degrees) by the line of Right Sines, do thus. First, Set one foot of your Compasses in the beginning of the line of Sines, and extend the other to the Sine of 30 degrees, with this extent set one foot of your Compasses in the Angular point at B, and with the other strike an Arch of a Circle, cutting the Line AB produced to a sufficient distance. Then taking 45 degrees from the beginning of the line of Sines in your

your Compasses, which is half of 90, (or the half of any other Angle required;) and setting one foot of your Compasses in the point of intersection where the Arch cutteth the Line AB, with the other foot of your Compasses make a mark in the said Arch, and from that mark draw a right line to the Angular point B, so is the Angle included double to the distance upon the Arch, *viz.* 90 degrees. And what is here said concerning the protracting of an Angle of 90 degrees, is in all Respects to be understood of any other Angle, or any Number of Degrees whatsoever, whether more or less, the same Rule of working being observed for one as well as the other.

And so much shall suffice to have spoken to the first particular concerning a Square, to wit, the Producing or Making of it.

C H A P.

C H A P. VII.

In the next place, I shall shew you the Reducing of it ; or rather, the Reducing of other Figures to it, which take as follows.

S E C T. I.

To Reduce a Long Square to a Geometrical Square.

WHich is no more but to find a mean proportional betwixt the length and the breadth of the Long Square, for that shall be the length of a side of a Geometrical Square equal in content thereunto. As suppose the length of the long Square to be 64, and the breadth 16, between which two the mean proportional is 32 ; so that a Geometrical Square, whose sides are 32, will be equal to this long Square.

Now to find a mean proportional by *Common Arithmetick*, or *Arithmetically*, is taught before by Logarithmes, (pag. 33.) By the Lines of Numbers, Sines, and Tangents, (pag. 43.) By Protraction, (pag. 60) where you have the variety of
doing

doing it all these several ways, to which I refer you.

S E C T. 2.

To Reduce a Rhombus or Rhomboides to a Geometrical Square.

From one of the Obtuse Angles let fall a perpendicular, and a mean proportional between the perpendicular and the side whereon it falls is the side of a Square equal to the same.

S E C T. 3.

To Reduce a Triangle to a Geometrical Square.

Is to find a mean proportional between half the Base and the Perpendicular, which shall be the side of a Square equal to the Triangle: As suppose the Base 72, half whereof is 36; and suppose the perpendicular be 25, the mean proportional will be 30. And so any other irregular Polygon, (or many-sided Figure) may be reduced into a Square, if first you Reduce the same into Triangles.

And thus much for Producing and Reducing a Geometrical Square: Now in the last place, to Measure it.

C H A P. VIII.

The last thing that remains to be spoke to about a Square, is the Measuring of it; of which I come now to Entreat.

S E C T. I.

To Measure the Square.

WHich is very easie and short, Multiply one of the sides in it self, or by it self, and the Product is the Superficial Content or Area: As 6 by 6 produceth 36, and 4 by 4 16, &c.

S E C T. 2.

But besides this Superficial, or Flat Square, there is also a Solid Square; such is a Cube; when the Top and the Bottom as well as the 4 sides are all Squares in fashion of a Dye. Now to Measure this.

First, Multiply the length by the breadth, that gives the Superficial Content, and that Superficial Content multiplyed again by the Depth, gives the Solid Content in Cube Inches.

C H A P.

C H A P. IX.

Of a Triangle.

HAVING spoken to the two former Geometrical Figures, the Circle and the Square. In the third and last place I come now to speak of a Triangle, which is the most admirable of all the Three; The Producing of which, the Reducing whereunto, and the Measuring whereof is the Sum and Substance of all the Mathematicks.

For hereby we measure both Sea and Land, all Regular, and all irregular Bodies; the very Heavens themselves, and all the Heavenly Bodies: All Real and Imaginary Magnitudes, Heights and Distances, whether accessible, or inaccessible, as in the following part of this Discourse you may best discern, when we come to particular Application of a Triangle, to Surveying, Navigation and Astronomy.

Triangles are of Two Sorts: Either 1. Plain; or, 2. Sphærical, each consisting of three Sides and three Angles, which together make six several Parts: Any three of which Parts being given, any, or all of the

the other three may be found out by the Golden Rule, or Rule of Proportion.

For Sphærical Triangles, they properly belong to Astronomy, and the Resolution of Astronomical Questions; therefore I shall pass them by in this place, and refer the Reader till I come to speak of Astronomy, and shall then shew you the Resolution of them; and how those Questions about which they are conversant may be resolv'd Instrumentally, with far more ease, more pleasure, and more plainness. At present I shall speak only to Plain or Right Lined *Triangles*, which are sufficient for *Surveying* and *Navigation*; for taking Heights and Distances both by Sea and Land.

Now of these, If any one of the Angles be a Right Angle, or 90 Degrees, it is called a Right Angled *Plain Triangle*. If any of the Angles be an *Obtuse Angle*, or more than 90 Degrees, it is called an *Obtuse Angled Plain Triangle*. If all the Angles be Acute less than 90 Degrees (which they must be if they be neither of the former,) then it is called an *Acute Angled Plain Triangle*. And of these in my former Method as followeth.

C H A P. X.

1. **I**N the first place I shall therefore shew you, how to Produce, Protract, or Frame a Triangle.

2. I shall shew you how all other Figures, whether Regular or Irregular may be Reduced to one Triangle, or however to many, and so measured by its Rules.

3. How to Measure a Triangle, or to find the Contents either of a Superficial, or of a Solid Triangle. And likewise by some of the six parts given, (*viz.* Sides and Angles) To find the rest, which is properly called the Resolution of a Triangle, or Trigonometry.

First then, To Produce, Protract, Lay down, or Frame a Triangle, is no more but the protracting or laying down of Lines and Angles. The Lines or Sides are laid down from a Line of Lines, or equal Parts, and the Angles are protracted by a Line of Chords, or by a Line of Sines, or a Protractor, as you see is taught in the making of a Square, *pag.* 83, 84, 85, &c. But because there is only mention made of a Right Angle, (a Square admitting no other) I shall here

here give you Examples of other Angles also. Now you must know that every Acute-angled Triangle, whose Angles are all Acute, doth contain 180 degrees, all the Angles added together; A Right Angled Triangle, besides the Right Angle, which is always 90 degrees, contains other 90 in the two other Angles; the one whereof is always the complement of the other to 90 degrees more, being in the whole 180 degrees, or a Semicircle exactly. An Obtuse Angled Triangle, by how much the more the Obtuse Angle is more than 90, by so much the other two are less than 90: All three together notwithstanding making exactly 180 degrees as the former.

So that the Three Angles of every Triangle are equal to two Right Angles, or 180 degrees, whether it be an Acute, Obtuse, or Right Angled Triangle.

As the Angles are thus distinguished into Right, Obtuse and Acute, so also are the Sides of a Right Angled Triangle (which is of most use and excellency) distinguished into these three, the Base, the Perpendicular, and the Hypothenuse, (In other Triangles, either Obtuse, or Acute, they are called the Hypothenuse, and the 2 Catheti.) Or others divide them into the subtending side, which is the side opposite to any Angle, and the two containing sides which

which are the sides next on either side the said Angle.

In a Right Angled Triangle, the Hypothenufe is always the side opposite to the Right Angle, the Base is usually accounted the longer of the other two, and the perpendicular the shorter, including the Right Angle betwixt them.

Now to make a Triangle, whose Base is 400 Inches, Feet, or Yards, and perpendicular 231, and Hypothenufe 462; the Right Angle 90 degrees, the Angle at the Base 30 degrees, and consequently the other Angle at the top of the perpendicular 60 degrees, being the complement of the other Angle at the Base to 90 degrees. Take this following Example.

First, Draw a line at pleasure AB, and with your Compasses take 4 of the larger equal parts from your Scale or Line of equal parts, and mark that distance upon the said Line AB, signifying 400, then upon the Point B raise a Perpendicular or protract an Angle of 90 degrees (which is all one) as is taught *pag.* 86. either by a Line of Chords, a Line of Sines, or Protractor. Then with your Compasses take 231 from the Line of equal Parts for the length of the perpendicular; and setting one Foot in the Angle at B, make a mark with the other for the length of the perpendicular. Then upon

upon the Angle A at the Base protract an Angle of 30 degrees on this manner.

S E C T. 1.

First, by a Line of Chords, set one foot of your Compasses at the beginning of the Line, and extend the other to 60 Degrees for the Radius: Then with this extent setting one foot in the point A, describe the Arch DE, cutting the Base in E, and then setting one foot of your Compasses again in the beginning of the Line of Chords; extend the other to 30 degrees, (that being the number of degrees the Angle is to contain) and setting one foot of the Compasses in the point E, where the Arch does intersect the Base; with this extent of 30 degrees make a mark, then laying your Ruler to the said mark, and the Angular point A, draw a line at length, or till it meet with the perpendicular, so shall the Angle at the Base contain 30 degrees, and the other at the Perpendicular 60, being the complement to 90 degrees.

S E C T. 2.

Now by the Line of Sines to protract the said Angle of 30 degrees. First, take in your Compasses 30 degrees of the Line of

F

Sines

lines, and at that distance draw the Arch, and at the distance of 15 degrees which is half of 30 the Angle required, draw the Hypothenuſal, ſo ſhall it be an Angle of 30 degrees.

S E C T. 3.

And laſtly, To protract this Angle of 30 degrees by the Protractor, do thus: Prick the Pin through the Center hole in the point A, and lay the Meridian Line upon the Baſe AB, and making a mark by the edge of the Protractor againſt 30 degrees, by the ſame mark, and the point A, draw the Hypothenuſal AC.

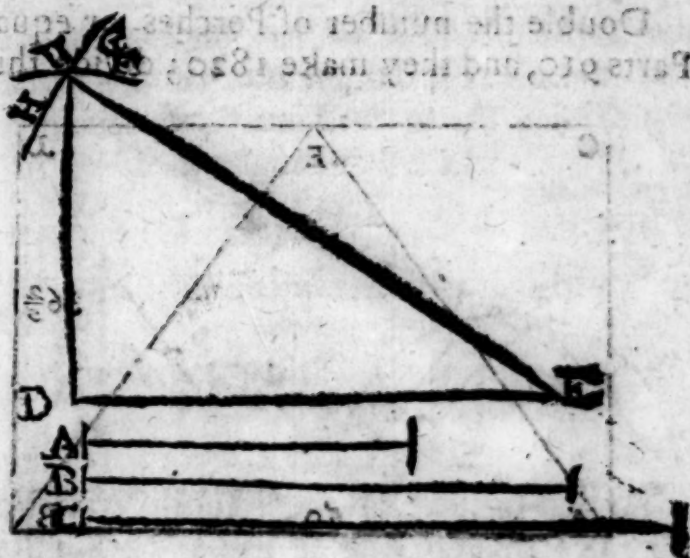


So that by any of theſe three ways you may protract an Angle, and any other Angle as well as this in the Example; obſerving the ſame Rules, which together with the

the sides protracted from a line of equal parts, makes up the compleat Triangle, ABC; and by consequence any other Triangle whatsoever: Only in an obtuse Angle, where it is more than 90 degrees, as suppose 140, you must first prick out 90 degrees, and from thence the remainder that is 50 degrees more, which together make 140: And the same Rule is to be observed for odd Minutes, as is here given for degrees all along.

S E C T. 4.

Three Lines being given, so that the two shortest together be longer than the Third, to make thereof a Triangle.



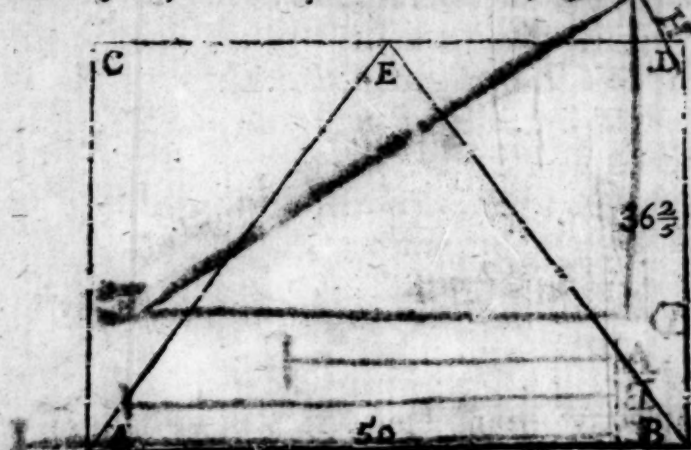
And suppose the Lines given be A, B, C,
First draw the line DE equal to the line B,
then take with your Compasses the line C,
F 2 and

and setting one foot in E, with the other describe the Arch HG: Also taking the given Line A in your Compasses, and placing one foot in D, with the other describe the Arch HE, cutting the former Arch HG in the point H. Lastly, if from the point H you draw the Lines HE and HD, you shall constitute the Triangle HDE, whose sides shall be equal to the three given Lines, ABC.

S E C T. 3.

To make a Triangle to contain 910 equal parts, (suppose Perches) whose Base shall be 50 of the same equal parts, be they what they will.

Double the number of Perches or equal Parts 910, and they make 1820; divide this



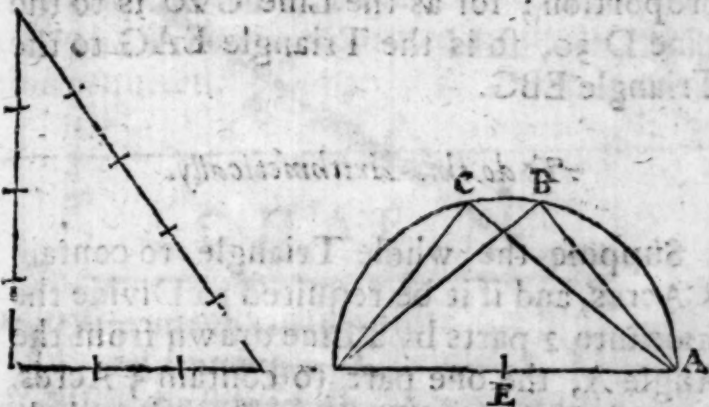
by the Base 50, the Quotient will be the length of the perpendicular of the Triangle,

gle, to wit, $36\frac{2}{3}$. Then from any Scale of equal parts lay down AB the Base 50, then upon B raise the Perpendicular BD $36\frac{2}{3}$; and draw the line CD parallel to AB: Then from any point in the line CD (as from E) draw the lines EA and EB, including the Triangle EAB, which shall contain 910 equal parts as required. And the like, if you would make a Triangle to contain any other number of Perches or equal Parts, the Rule is the same *mutatis mutandis*.

(82.) *SECT. 6.*

To make a Right Angle readily.

Take a Line for the Hypotenuse, and divide it into five equal parts, then take



three of those equal parts for the Base, and four for the Perpendicular, and joyn these

F 3

toge-

together : Or, draw a Semicircle, as ABCD upon the Center E, (DA the Diameter) and from A draw a Line to the Circumference, as at B or C : Then from B or C draw Lines to A or D, so have you made a right Angle, as ABD, or ACD.

S E C T. 7.

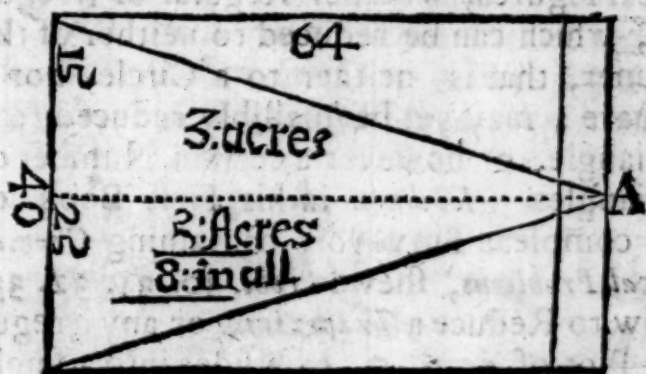
To Divide a Triangle into any proportion whatsoever.

By the Fifth Geometrical Problem, (p. 58.) Divide the Base of the said Triangle in proportion, as 20 is to 30, as there it is done, and from the point of Division at G draw a Line to the opposite Angle at E, so is the Triangle it self divided in the same proportion ; for as the Line C 20 is to the Line D 30, so is the Triangle EAG to the Triangle EBG.

To do this Arithmetically.

Suppose the whole Triangle to contain 8 Acres, and if it be required to Divide the same into 2 parts by a Line drawn from the Angle A, the one part to contain 5 Acres, the other three. First measure the whole length of the Base, which suppose 40, then say by the Rule of Proportion. If eight Acres
(the

(the Quantity of the whole Triangle) give
40 (the whole Base) what parts of the



Base shall five Acres give? Multiply and Divide, the Quotient will be 25 for the greater Segment, which being deducted from 40 (the whole Base) there will remain 15 for the lesser Segment, then draw a Line from the opposite Angle, which shall Divide the Triangle according to the proportion required.

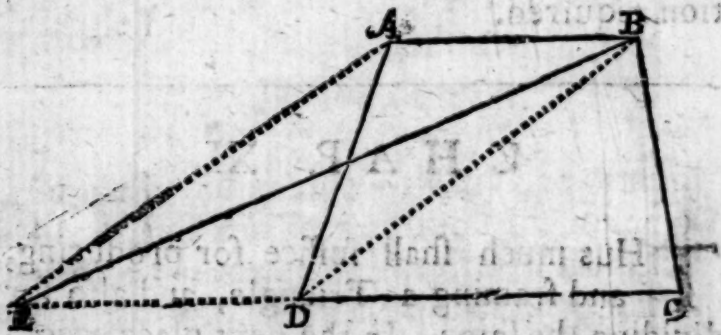
C H A P. XI.

THus much shall suffice for producing, and framing a Triangle, and also for dividing the same: In the next place according to my former Method, to shew how other Figures may be Reduced to Triangles:

gles: And herein appears the excellency of a Triangle above the rest, that all Geometrical Figures, whether Regular or Irregular, which can be reduced to neither of the former, that is, neither to a Circle, nor a Square; may yet be infallibly reduced to a Triangle, or however a certain Number of Triangles. *Leyburn* in his First Book of his compleat Surveyor, containing *Geometrical Problems*, shews *Probl. 30. 31. 32. 33.* How to Reduce a *Trapezium*, or any irregular Plot of 5, 6, 7, or 8 sides into a single Triangle, which I abridge in this manner.

To Reduce a Trapezium into a Triangle containing 4 unequal sides.

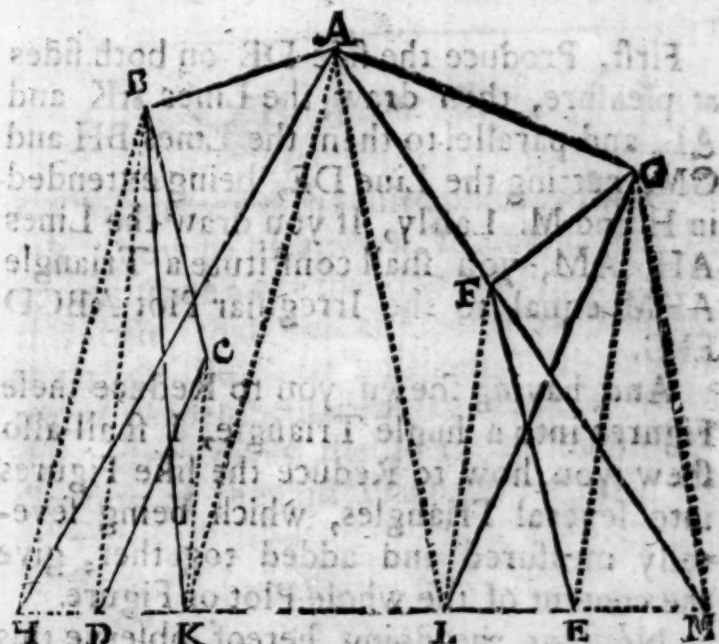
Extend the line DC, and draw the Diagonal BD. Then from the Point A draw



the line AE parallel to BD, extending it till it cut the side CD in the point E. Lastly,
From

From B draw the line BE, constituting the Triangle EBC, which shall be equal to the Trapezium ABCD.

How to Reduce an Irregular Plot of 5, 6, 7 Sides into a Triangle.



Let ABCDEFG. First Reduce this Figure of 7 sides into a Figure of 5 sides, keeping the same quantity, ABKLG on this manner. Draw the line BD, and parallel thereunto CK, then draw the line BK, whereunto the two lines BC and CD are thereby reduced. Then draw the line GE, and parallel thereunto FL, then if you draw the line GL,

the two sides GF and FE shall thereby be reduced to one straight Line, viz. GL, and the whole Plot to a Figure of five sides ABKLG.

Now to Reduce this to a Triangle.

First, Produce the side DE on both sides at pleasure, then draw the Lines AK and AL, and parallel to them the Lines BH and GM, cutting the Line DE, being extended in H and M. Lastly, If you draw the Lines AH, AM, you shall constitute a Triangle AHM equal to the Irregular Plot ABCDEFG.

And having shewn you to Reduce these Figures into a single Triangle, I shall also shew you how to Reduce the like Figures into several Triangles, which being severally measured and added together, give the content of the whole Plot or Figure.

Now for the doing hereof observe this general Rule, That the Number of Triangles contained in any Figure or Plot, will be always two fewer, or not so many by Two as the number of Angles, or Sides which the said Figure or Plot doth contain.

As any Square or Trapezium which contains 4 Angles, and 4 Sides may be easily reduced to 2 Triangles drawn from Corner to Corner, so likewise a Plot of 5, 6, 7, or 8 sides

8 sides may be easily reduced to 3, 4, 5, or 6 Triangles, as will appear upon practice, and needs no Example. Now these Triangles measured particularly, and then the whole added together, gives the content of any Irregular Plot or Figure whatsoever.

C H A P. XII.

Therefore in the third place to shew you how to measure a Triangle, and by consequence any other Plot or Figure, take this Rule, that half the perpendicular and the whole Base, or half the Base, and the whole perpendicular multiplyed the one by the other, gives the Area or Superficial Content of the said Triangle.

S E C T. I.

Now the perpendicular of a Triangle is a Plumb-line let fall from any Angle upon its opposite side, which we call the Base: How to let fall this perpendicular from any Angle or Point you have *Probl. 3. pag. 55.* to which I refer you.

Now for an Equilateral Triangle, or a Triangle having three equal sides, observe this

this Rule: That the power of the side is to the power of the perpendicular let fall from any of the Angles to the subtendent side in proportion se squitertia, or as 4 to 3, or $\frac{4}{3}$ of the other: Note the Power of a Line is the Square thereof.

S E C T. 2.

Any three sides of any Triangle being known, to find the Perpendicular: The greatest side being assigned for the Base, upon which the perpendicular shall be supposed to fall: First find the Sum of the other two sides, (which suppose 13 and 11.) Then observe the difference betwixt these two sides, which is 2; that done, suppose the whole Base to be 20: Say by the Rule of Proportion, As the whole Base 20 is to the Sum of the other two sides added together, viz. 24, so is the difference of the other two sides, viz. 2, to a fourth number, which being deducted out of the Base, the perpendicular will fall in the middle of that which remains, 2, $\frac{2}{3}$ deducted from 20, rests 17 $\frac{2}{3}$, the half whereof is 8 $\frac{1}{3}$: a line drawn from that point from the Base to the opposite Angle, is the true perpendicular to the said Triangle.

Another way to find the Perpendicular of an Equilateral Triangle.

Multiply one of the sides by 13, and divide the Product by 12, the Quotient is the perpendicular, says Brown in his Triangular Quadrant.

C H A P. XII.

THUS much for measuring a Superficial Triangle, now a solid Triangle is that Figure, or Solid Regular Platonick Body called a Tetrahedron, contained under 4 equal and Equilateral Triangles, or a Triangular Pyramid: The solid content whereof is found by Multiplying the Area or Superficial content of the Base by one third part of the length of the Pyramid or Tetrahedron from the midst of one plain to the Apex or Top of the Solid Angle opposite thereunto.

Note that this Triangular Pyramid is little more than $\frac{1}{3}$ of a Cube of equal Base and Altitude: A Cylinder is $\frac{1}{4}$ of a Cube of equal Base and Altitude.

A Globe is $\frac{1}{2}$ of a Cube, or $\frac{2}{3}$ of a Cylinder, whose sides and diameters are equal.

How

How to Measure the Solid Content of any Body, how irregular soever it be, The Form and Fashion not regarded.

To perform this, you must prepare an hollow Cube, into which put your Irregular Body, which being placed therein, you shall pour in so much Water, till it no more than just cover the Body in the Cube, then make a mark in the inside of the Cube where the Superficies of the Water toucheth. This done, take but the irregular Body, and mark again directly under the former, where the Brim of the Water now touches; for the distance of these two marks multiplied by the Square of the Cubes side, produceth the Crassitude of that Irregular Body. For Example.

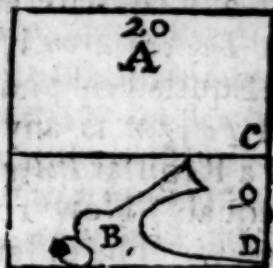
Suppose

BOOK II. Of Geometry.

III

Suppose A to be the Cubical Hollow

Vessel, whose inward side suppose to be 20 Inches, B the irregular Body, whose solid Content is required. First then I put B into the Hollow Cube A, and pouring in water till it be covered, admit the top of the water to reach to C: Then taking out the irregular Body again, admit the



20

20

400

9

3600 Inches.

Superficies of the Water fall to D, then measure the distance betwixt C and D, suppose 9 Inches, which Multiplied into 400, the Square of the Cubes side produceth 3600; and so many Cubical Inches are contained in the Irregular Body B. By this you may measure any Part or Portion of a Solid Body.

Now having spoken of a Solid Circle; viz. a Globe, and of a Solid Square, viz. a Cube: And now of a Solid Triangle, viz. a Tetrahedron. These two last being two of the five Platonick Bodies, I shall add the other three.

An Octahedron is contained under eight equal and equilateral Triangles.

An

An *Icosaedron* is contained under 20 equal and equilateral Triangles.

A *Dodecaedron* is contained under 12 equal Equilateral and Equiangular *Pentagons*.

A *Polygon* is any Figure of many sides, and a *Regular Polygon* is one whose sides are all equal. Now to measure any *Regular Polygon*. The distance from the Center to the middle of one side, and the half Sum of the measure of all the sides multiplied together, shall be the true Area or Content thereof. To measure any *Irregular Polygon*, see pag. 105, by Reducing it to a Triangle.

If the Area or Content be 100, then the Sides of these *Regular Figures* are as follows.

An Equilateral Triangle,	{ its perpendicular is 13	13
	{ its side	25
A Square its side.		20
A Pentagon, which is a Figure of 5 sides.		26
An Hexagon of 6 sides, its side		26
An Heptagon of 7 sides, its side		26
An Octagon of 8 sides, its side		26
A Nonagon of 9 sides		26
A Decagon of 10 sides		26
The Semidiameter or Radius of a Circle.		26

A Cone differs from a Pyramid, because a Cone hath always a round Base and Superficies like a Sugar-loaf: But a Pyramid hath an Angular Base and Superficies of several

veral sides : The way to measure both as to the Solid Content. First, find the Superficial content of the Base, and Multiply the same by the third part of a perpendicular from the sharp end to the Base. And according to these Proportions before mentioned is any other Quantity or content to be found out : As suppose I would have a Triangle to contain 200, what must the sides be ? The half distance in the line of numbers betwixt 100 and 200 will reach from 15.2 to 21.5 for its side, and from 13.123 the perpendicular of a Triangle, whose Area is 100 ; the same extent will reach to 18.6 the perpendicular of an Equilateral Triangle, whose Area is 200, &c.

S E C T. 3.

The three sides of a right-lin'd Triangle being given, to find the Superficial Content.

Add the three sides together, (as 20. 13. 11.) which make 44, the half of which is 22 ; from which half Sum subtract each side severally to the end you may have the difference betwixt that half Sum and each side, which will be 2. 9. 11. Then add the Logarithmes of the said half Sum and of those differences together, and lastly, Divide the Sum of all those Logarithmes

garithmes by 2, so you will have the Logarithme of the Superficial Content or Area of the Triangle.

The half Sum	22.	1.342423
The Differences	2.	0.301030
	9.	0.954243
	11.	1.041393

The Sum of the Logarithmes — 3.639089
 The Area or Content req. 66. 1.819544

OF

OF
GAUGING
ALL SORTS OF
VESSELS.

Gauging of Vessels is no more but Measuring of Solids. These Solids are to be Reduced to some Regular Geometrical Figure (as is taught before when we treated of Measuring of Solids in General) especially to one of these Three.

1. The Solid Square or Cube.
2. The Solid Triangle, or any Irregular Solid.
3. The Solid Circle or Cylinder.

I. To

1. *To Measure a Solid Square, or any Vessel in the form of a Cube.*

The Rule is this.

Multiply the Length by the Breadth, and that Product by the Depth (in Inches) and that gives the contents in Square Inches.

Now to bring these Inches into Ale, Wine, or Corn Gallons.

Observe this Rule once for all, That formerly the Ale Gallon was accounted to contain $288 \frac{1}{4}$ Cube Inches. But by the care and pains of Mr. *Nicholas Gunton*, the just quantity of the Quart for Ale remaining in the hands of the Chamberlains of His Majesties Exchequer appears to be $70 \frac{1}{2}$ Square Inches, which makes the Gallon 282, as some have of late found to their no small cost. And I am of opinion (saith *Mayne*) that if the Wine Gallon were carefully examined, (and so also the Corn Gallon) they would prove to contain less than they are commonly holden to do.

But to proceed, for Ale divide the Product in Square Inches by 282; for Wine by 231, and for Corn by $272 \frac{1}{4}$, and the Quotient of each is the Gallons; and if any

any Fractions remain to Reduce it into Pints, multiply the Numerator by 8, (suppose $\frac{128}{282}$ be the Fraction) that is 128 by 8, and Divide the Product by the old Denominator 282, the Quotient shews the Pints. Thus much for measuring the Square or Cubical Vessel, you may also find pag. 89, 90, what other Figures may be reduced to this, as a Long Square, a Rhombus, or Rhomboides, a Triangle, a Circle, with its Segments, &c.

2. Now in the Second place of Measure, a Solid Triangle, for any Vessel of a Triangular Form.

The Rule is,

Multiply the perpendicular by half the Base, or the whole Base by half the perpendicular, and divide the Product by 282, the Quotient will be the Ale Gallons contained in one Inch of Depth upon that Triangle. If the Vessel be of an Irregular Form, Divide it into several Triangles, and let fall perpendiculars in each, and so find their several Areas and Contents, and then add them together. So have you the Content or Area of the whole Figure, be it never so Irregular.

3. In the third place to measure any Vessel which is of a Circular or Cylindrical Form; That is to say, Either Cylinders, Or, to be Reduced to Cylinders.

The Rule is this,
Multiply half the Diameter by half the Circumference, gives the Area or Superficial Content of the Base or Bottoms, and then Multiply that Product by the Depth, gives the Solid Content in Inches, which Square Inches you may bring into Gallons, and Pints, as before is taught.

To find the Diameter of any Circle by the Circumference.

Multiply the Circumference by 7, and Divide the Product by 22.

To find the Circumference by the Diameter.

Multiply the Diameter by 22, and Divide the Product by 7.

To find the Area or Superficial Content of a Circle by the Diameter.

Square the Diameter, that is, Multiply it by it self, then Multiply the Product by

by 11, and divide by 14, the Quotient gives the Area or Superficial Content: For as 14 is to 11, so is the Square of the Diameter to the Area, or at least the nearest Approximation to it for Common Practice, or which comes infinitely near the Truth. As Unity is to 3.14159, so is the Square of the Semidiameter to the Area in Square Inches. Or,

Take this short Rule to find the Ale or Wine Gallons in any Circle by the Diameter, and one Inch in depth.

Square the Diameter, (that is, Multiply it by it self,) and divide that Square or Product by 359 for Ale Gallons, or by 294 for Wine Gallons; and then Multiply that Area or Superficial Content by the Length or Depth of the Vessel, and that will give you the Solid Content of the whole Vessel in Gallons.

Thus far we have supposed the Vessel to be measured to be perfectly Circular or Cylindrical; and the Top and the Bottom to be both of the same Diameter: But if the Top and the Bottom be of different Diameters, (or which is the same) the Head and the Bung, then you are to find a Mean Diameter.

Which

Which when the Vessel is a perfect Cone, and the sides strait.

First, Find the Area of the Base or Bottom, and multiply that Area by the third part of the Altitude, the Product is the Solidity of the whole Cone.

If the Vessel be not a whole Cone, but only a Part or Frustum of a Cone.

Add the Diameters at Top and Bottom together, and take the half of that Sum for the mean Diameter.

To Measure a Vessel, which is an Irregular Cylinder (such as a Butt, Pipe, Hoghead, Barrel, &c.) find the Mean Diameter by these Rules.

1. Add double the Bung Diameter to once the Head Diameter, and Divide their Sum by 3, the Quotient take for your mean Diameter.

2. More exactly, To the doubled Square of the Bung Diameter, add the Square of the head Diameter, and that Sum Multiply by the Cask length, the last Product Divide by 1077, the Quotient is the Ale Gallons; or by 882 Quotes the Wine Gallons contained in that Cask.

3. By

3. *By Mr. Oughtred's way of Measuring the Frustrum of a Spheroid.*

First, take the Diameter at the Bung, and find the Area of the Circle answerable thereunto, and take two thirds of that Area. Secondly, Take the Diameter at the Head, and find the Area of that Circle, and take one third of that Area; add these two Sums together, and multiply the whole by the length of the Vessel, the Product gives the content of the whole Vessel in Cubique Inches, which divided by 282 for Ale, or 231 for Wine, gives the Gallons.

To Measure a Vessel, which is an Irregular Cylinder, such as a Butt, Pipe, Hogshhead, Barrel, or the like.

Take the Diameter at the Bung, which suppose 23 Inches, and then take the Diameter at the Head, which suppose to be 20, the difference is 3: Now (in respect to the bending of the Staves in proportion, as 7 to 10) Multiply this difference 3 by 7 makes 21, and divide the Product by 10, makes $2\frac{1}{5}$, which added to the lesser Diameter 20, makes $22\frac{1}{5}$ for the Mean Diameter: Half of this Mean Diameter multiplied by half of its proportional Circumference,

G

ference, gives the Area or Superficial Content, and that multiplied by the Length of the Vessel, gives its Solid Content in Inches, and those Inches reduced to Gallons by dividing by 282 for Ale, and 231 for Wine, *prout supra*.

To find a Mean Diameter by Gunter's Line of Numbers.

Extend the Compasses from the Gauge Point to the mean diameter, the same Extent (twice repeated) will reach from the Length of the Cask, to the whole Content of the Vessel. This Gauge Point is at 17.2 for Ale, almost 19 for Wine.

Mr. *Mayne* (in his little Book, called his *Practical Gauger*, which I recommend as the plainest, and easiest, and shortest I ever met with,) besides very good Rules for Gauging all sorts of Vessels, and the use of the Gauging Rule, which is very well done. He hath (besides these) 4 very useful Tables, which I shall here mention, but refer the Reader to the Book it self.

1. The First is a Table of Areas of Circles in Ale Gallons and Millesimal Parts; to every Quarter of an Inch, from 10 to 14.4 Inches Diameter. This is to find the Contents of the Frustum of a Spheroid in Ale Gallons, *pag. 29.*

2. A Table

2. A Table of one Thirds of Area's of Circles in Wine Gallons, calculated to every Quarter of an Inch, from 10 to 60 Inches Diameter, *pag.* 39. This Table is to find readily the Contents of a Wine Hoghead, or Beer Barrel in Wine Gallons.

3. A Table of the Contents of Cylinders in Ale Gallons, and Centesimal Parts from 12 to 60 Inches Diameter, and to 8 Inches in Depth ready cast up, so that you have nothing to do but to find the Plain Diameter, or the Mean Diameter of any Cylindrical Vessel.

4. In the Fourth place, A Table of Area's of Segments of a Circle: The Radius divided into 100 parts, calculated to the $\frac{1}{10000}$ part of a Square Inch. Hereby you may find the Vacuity, Ullage, or Wants in a Cask partly full, lying with his Axis parallel to the Horizon: The Cask being taken as the Frustum of a Spheroid: The Table and use of it follows.

A TABLE of Area's of Segments.

V. Area	V. Area	V. Area	V. Area
1.0017	26.2066	51.5127	76.3155
2.0048	27.2178	52.5255	77.8263
3.0087	28.2292	53.5382	78.8369
4.0134	29.2407	54.5509	79.8473
5.0187	30.2523	55.5636	80.8576
6.0245	31.2640	56.5762	81.8677
7.0308	32.2759	57.5888	82.8776
8.0375	33.2878	58.6014	83.8873
9.0446	34.2998	59.6140	84.8967
10.0520	35.3119	60.6265	85.9059
11.0598	36.3241	61.6389	86.9149
12.0680	37.3364	62.6513	87.9236
13.0764	38.3487	63.6636	88.9320
14.0851	39.3611	64.6759	89.9402
15.0941	40.3735	65.6881	90.9480
16.1033	41.3860	66.7002	91.9554
17.1127	42.3986	67.7122	92.9625
18.1224	43.4112	68.7241	93.9692
19.1323	44.4238	69.7360	94.9755
20.1424	45.4364	70.7477	95.9813
21.1527	46.4491	71.7593	96.9866
22.1631	47.4618	72.7708	97.9913
23.1737	48.4745	73.7822	98.9952
24.1845	49.4873	74.7934	99.9983
25.1955	50.5000	75.8045	100.

The Use of this Table is as follows.

1. It is Requisite that the Bung and Head Diameters, the Cask Length, the whole Content, and the Dry and Wet Inches be all known, and then if the Question be

What is wanting, or what is remaining in the Cask?

Divide accordingly either the Dry or Wet Inches by the Bung Diameter, and the Quotient seek in the Table under V, or Versed Sine; against it stands a Number, which multiplyed by the whole Content exhibits the vacuity, if your Dividend were the dry Inches; or shews the remaining Liquor, if your Dividend were the wet Inches.

Suppose the Bung Diameter 28, the Content of the Cask 60 Gallons, and Dry Inches 7.

Divide 7 by 28, by adding two Cyphers,

$$\begin{array}{r} 700 \overline{) 28} \\ 288 \\ \hline 2 \end{array}$$

seek this 25 in the Table, over against it you find 1955, which Number multiply by the whole Content 60, so is the Ullage or Wants, 11.7300; cutting off the last

G 3

4 Figures

4 Figures is 11 Gallons, and almost $\frac{1}{4}$ of a Gallon.

Now if the Question be, What Quantity of Liquor is remaining in this Cask?

Divide the Wet Inches by the Bung Diameter, that is 21 the Wet Inches by 28 thus, $28 \overline{) 2100} 75$. Now against 75 in the Table you find 8045, which being multiplied by 60 the whole Content of the Cask.

The Remaining Liquor is	48.2700
The Wants was	11.7300

The whole Contents	60.0000
--------------------	---------

Now if after Division there happen a Remainder or Fraction, and that be above Half the Divisor, I take the next bigger Number: Or if it be less than half the Divisor, I take the same Number which is in the Quotient.

The Description and Use of the Gauging Rule.

This Rule is commonly 4 Foot long, and is made to double in 4 joints for convenient Portage: It hath also 4 sides, on which are drawn several Lines.

1. There

1. There are two Lines called Diagonals the one for Ale, the other for Wine Measure; put the end which is cut slope-ways in at the Bung-hole, and let it touch the bottom of the Head; and the Number at the Bung shews the Ale or Wine Gallons respectively: This will give a very good estimate of the Content of all Cask in the form of a *London* Beer-Barrel, or the *French* Wine Hoghead.

2. On another side there is put a Line of Inches from 1 to 48, each Inch decimally divided: And also upon the same side you have *Oughtred's* Gauge Line, it being a Line of one Thirds of Area's of Circles in Wine Gallons, by which you may Gauge a Cask after this manner.

Put your Rule down at the Bung perpendicularly, observing what Number appears just even with the inside of the Cask, suppose it 7: set that down twice, then take the Diameter at the Head, and let that shew you 6 upon the same Line, let that down to the former, add these three Numbers together, and multiply the Sum by the Cask length suppose 30, then cut off one place from the Product towards the Right Hand, and the Figures towards the Left Hand are your Number of Wine Gallons contained in that Cask.

If your Diameter fall amongst the Divisions between the Numbers, you must cut off 2 places from the Product. *Example,*

Bung Diameter	71	7	
	71	7	
Head Diameter	58	6	
	200	20	
Cask Length.	30	30	Length.
Content	60.00	60.0	Content.

3. On a third Face of this Rule is a Line of equal Parts numbred from 1 to 96; This Line considered together with that last before-mentioned, (*viz.* *Oughtred's* Gauge-Line) do make a Table of Area's of Circles in Ale-Gallons; so that if you find your Diameter in this Line, turn up the other Face, and against your Diameter you shall have the Area of your Circle in Ale-Measure. As for *Example.*

The Diameter of a Circle is 19 Inches, the Area of that Circle upon the other edge in *Oughtred's* Line is a little above one Gallon, so the Diameter 30 Inches, the Area is 2.5 Gallons, so the Diameter 67 inches, it holds 12.5 Gallons upon one Inch of Depth.

The use of these Lines thus together is
the

the same with that of the Table of Areas of Circle in Ale Gallons, *pag.* 124.

4. On the fourth side of this Rule is drawn a line of Numbers, vulgarly called *Gunters Line*, being a Line of Logarithmes, whose use I have described in *Instrumental Arithmetick*, *pag.* 37, 38, 39, 40: Only in this place take notice as to Gauging, that at 17.2 you have a small Brass Pin, whereon to set the foot of your Compasses, and is called the Gauge-point for Wine Gallons marked *W. G.* And another small Brass Pin at almost 19 the Gauge-point for Ale Gallons, marked *A. G.* by which they are easily known. Now to Gauge a Cask by this line, you must first find the Diameter at the Head and Bung, and also the Casks length by the line of Inches: These being had find your mean Diameter, by adding double the Bung Diameter, to once the Diameter at the Head, and divide their Sum by 3, the Quotient take for your Mean Diameter: Then with your Compasses set one foot in the Gauge-point, whether it be for Wine or Ale respectively, and Extend the other to the mean diameter upon your Line of Numbers, and keeping the points at that distance, set one foot at the Number, expressing the Casks Length, and from thence double the distance of the feet of Compasses, exhibits the Content in Ale

or Wine Gallons respectively : As for Example, Bung diameter 27, Head diameter 24, Length 30 Inches, the mean diameter will be 26, which in Ale Gallons 57, in Wine Gallons 69, and having the mean diameter of a Conical Tun ($28\frac{1}{2}$) depth of Liquor (29) The Quantity of Ale Gallons will be found ($63\frac{1}{2}$) after the same manner, and is an Approximation near enough the Truth for common practice.

§ There is also another Line that runs parallel with this Line of Numbers, and is called a Line of Segments, but I do not like the Hypothesis upon which it is framed; the way by the Table of Areas of Segments being more perfect; However it being upon the Rule, I shall shew you how to find the Ullage or Wants in a Cask by this Line and the Line of Numbers.

Suppose the Bung diameter 24 Inches, wet 18 Inches, Content 50 Ale Gallons, What is the Ullage or Wants in this Cask?

As 24 on the Line of Numbers is to Radius on the Segments, so is (6) the dry Inches on the Line of Numbers to 17.8 on the Segments. Then as Unity to 50 on the Line of Numbers, so is 17.8 on the Numbers to 8.9 Ale Gallons, the wants required. So much for Gauging.

C H A P.

C H A P. XIV.

NOW besides this kind of measuring a Triangle, by taking the whole Content both of a Superficial and Solid Triangle; there is another way of Measuring a Triangle; or of finding the length of some of its sides, or the Quantity of some of its Angles by other Sides and Angles given or known.

For every Triangle consisting of 3 sides and 3 Angles, any 3 of these 6 parts being known, the rest may be easily found, either first Geometrically by Protraction, pag. 85, 86, 87. Or Secondly, Arithmetically, by the Rule of Proportion, as it is performed Logarithmically, pag. 31. and Instrumentally, pag. 41. This I call properly the Resolution of a Triangle, or Trigonometry.

Many Learned Authors have made this very particular the subject of whole Books, as *Petiscus*, *Norwood*, and others; but for my present purpose, I shall content my self with these following necessary Rules and Directions, which well applied, contain in short the whole Art of Trigonometry.

O F

O F
TRIGONOMETRY.

S E C T. I.

First then, By *Protraction*, or by
Scale and Compass.

P R O B L E M I.

*Having any 2 Angles and a Side included or
contained betwixt them, to find the other
Two Sides, and the Third Angle.*

AS suppose the Angles given in the fol-
lowing Triangle were the right An-
gle at the Base 90 degrees, and the Angle
at the Hypothenuse 30 degrees, and the
Base included betwixt these two Angles
400. To find the length of the Hypothe-
nuse,

nuse, and the length of the Perpendicular, as also the third Angle at the Perpendicular.

First, From a line of equal Parts take in your Compasses the Length of the Base 400, and drawing a Line at pleasure, prick the said distance upon the said line: And at one end of the Base protract an Angle of 30 degrees DE: At the other end of the said Base protract another Angle of 90 degrees, as FG, and drawing the perpendicular infinitely, and also the Hypothenuse, where these two intersect, or cut one another, doth confine the length of both the sides measured upon the same Line of equal Parts with your Compasses; and also doth include the Third Angle sought betwixt them, *viz.* 60 degrees.

P R O B L. 2.

Having Two Sides, and the Angle included betwixt them, to find the other Two Angles, and the Third Side.

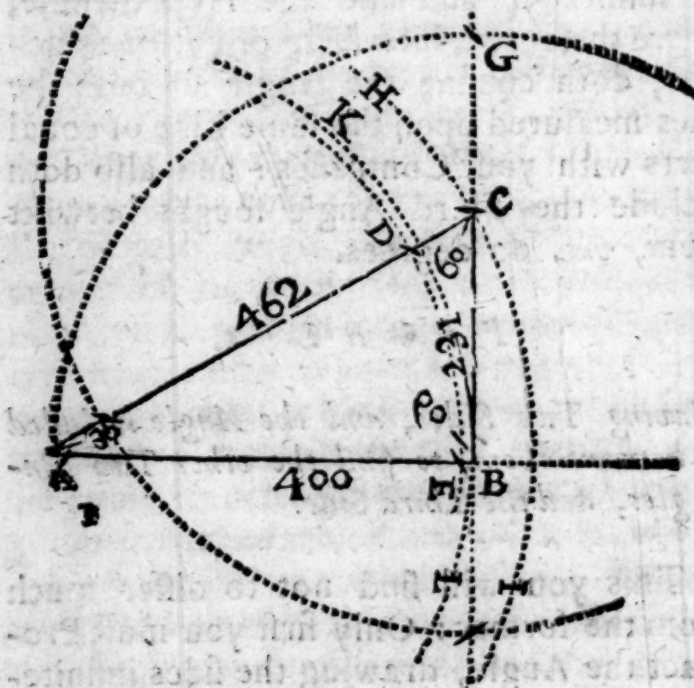
This you will find not to differ much from the former: Only first you must Protract the Angle, drawing the sides infinitely, and afterwards from the Angular point prick out either way their Exact Length, and from those Points draw the third side, which

which will include at either end the Two Angles sought.

P R O B L. 2.

Having the Three Sides given, to find the Three Angles of any Right Lined Triangle.

First, Take the distance of the longest side in your Compasses, and from any point as A, with that same distance draw an Arch of a Circle HI; then take the side



next the longest, and from the same point draw another Arch KL. Lastly, from any

any point in one of these Arches, to any other point in the other Arch, prick the length of the Third side, and drawing Lines from these three points from one to another, these Lines or Sides will include the three Angles required, A. B. C. A containing 30 degrees, B 90 degrees, and C 60 degrees.

This way of finding Sides and Angles by Geometry or Protraction, though it is not so exact as by Arithmetical Operation; yet it is sufficient for ordinary Use, and of all other, both the most easie and demonstrative, and also the most pleasant and delightful.

Note, That in every right Angled Triangle, if you have one of the acute Angles given, the other is also given, because it is the complement thereof to 90 degrees as the complement of 30 is 60. Also, if in any right Lin'd Triangle whatsoever you have any two of the Angles given, you have also the third Angle given, because the three Angles of any right lin'd Triangle, is equal to a Semicircle, or 180 degrees.

S E C T.

S E C T. 2.

But in the Second Place, by some sides or Angles of a Triangle given, to find the Rest, (which is properly the Resolution of a Triangle) is the most exactly performed by Arithmetick.

P R O B L. 1.

First, By common Arithmetick, Sides may be found by these Rules.

The Square of the Base and Perpendicular is equal to the Square of the Hypothenuse: As suppose the Base to be 4, the Square of 4 is 16; then suppose the perpendicular 3, the Square of 3 is 9; these 2 Numbers added together is 25: Now the Hypothenuse being 5, the Square of 5 is also 25. Likewise having the Hypothenuse, and one of the other sides, you may find the other side: As suppose the Hypothenuse 5, the Square of which is 25; as 5 multiplied by 5 is 25: Then suppose the other side given be the Base 4, the Square of that multiplied in it self is 16: This being deducted from the Square of the Hypothenuse 25, there Rests 9, which is the Square of the perpendicular 3: The same Rule to be obser-

observed if the Perpendicular had been given, and the Base required.

S E C T. 3.

Having shewed you how to find Sides by *Common Arithmetick*, I shall proceed to shew you, how both Sides and Angles may be found by *Logarithmetical* and *Instrumental Arithmetick* by these following Rules. The first is more particular for Right Angled Triangles only, the other two more general for all plain Triangles whatsoever.

R U L E I.

1. First then, having Two Sides, and a Right Angle betwixt them given, to find either of the Two Angles, and the Third side. The Proportion is,

As the greater side given is to the lesser side, so is the Tangent of 45 degrees to the Tangent of the lesser Angle, and its complement to 90 is the greater Angle: As for Example, in the former Triangle, pag. 134: The Base being given 400, and the perpendicular 231, and the Right Angle 90 contained betwixt them: Then say by Logarithmes, as the Logarithme of 400 is to the Logarithme of 231, so is the Tangent

Tangent of 45 to the Tangent of the lesser Angle, (*viz.* 30 degrees) which deducted from 90, leaves 60 degrees for the other Angle: This is done by working by the *Rule of Three* in *Logarithmes*, as is taught *pag.* 31. but I shall give you another Example in this place.

If 400 give 231, what gives Tangent 45?

The Logarithme of 400 is 2.602059

The Logarithme of 231 is 2.363612

The Tangent of 45 is 10.000000

These 2 added together is 12.363612

From whence deduct the first 2.602059

Rests the Tangent of 30 deg. 9.761553

for the lesser Angle: The complement whereof to 90 degrees, is 60 the greater Angle.

The same may be done by Instrumental Arithmetick, or the Lines of Numbers and Tangents by the Rule of Three upon those Lines as is taught *pag.* 41; but I shall here add this Example more, Extend the Compasses from 400 to 231 in the Line of Numbers, the same Extent upon the Line of Tangents.

Tangents will reach from 45 to 30 degrees for the lesser Angle : the greater Angle being its complement to 90 degrees, viz. 60 : And the two Angles found, and the two sides given, you may easily find the third side by the Rule in *Common Arithmetick*, pag. 136, or by the third Rule following, to which I refer you.

R U L E 2.

The second Rule for resolving of a Triangle is more general than the former, because it resolves not only a right Angled Triangle, but also an Acute, and an Obtuse Angled Triangle, that is any Right Lin'd Triangle whatsoever, and it is this.

As the Sum of two of the sides is to the difference of the same sides, so is the Tangent of half the Sum of the opposite Angles to the Tangent of half their difference.

Which I shall more fully explain by this Example, As suppose two sides to be given, and any Angle betwixt them, to find either of the other Angles : First, Add the sides together, which suppose 303 and 176 makes 479 : And Secondly, Subtract the one from the other, their difference is 127. Now suppose the Angle given to be 110 d. 30 m.
Take

Take the complement thereof to 180, which is 69 d. 30 m. and this is the Sum of the 2 unknown Angles, (because every right lin'd Triangle is equal to 2 right Angles, or 180 degrees.) Now the half of this 69 d. 30 m. is 34 d. 45 m. this being done, the Proportion is,

As the Logarithme of the whole Sum 479 is to the Logarithme of the difference 127, to is the Tangent of the half Sum of the two unknown Angles, viz. 34 d. 45 m. To the Tangent of half the difference between the said Angles, viz. 10 d. 25 m.

For the said 10 d. 25 m. being added to the half Sum (34 d. 45 m.) the Sum will be 45 d. 10 m. the quantity of the bigger Angle; and being subtracted from the same, will shew the Quantity of the lesser Angle, viz. 24 d. 20 m.

The Proportion may be wrought either by the *Rule of Three* in Logarithmetical; or by the Lines of Numbers and Tangents in *Instrumental Arithmetick*, pag. 31 and 41.

R U L E 3.

The Third General Rule, which is the last, I shall lay down, but not the least, is the most general of all the rest, and indeed may serve instead of all; and by me has been accounted even all in all; inasmuch as I once thought to have given no other, and but only for varieties sake there is no other abso-

BOOK II. Of Trigonometry.

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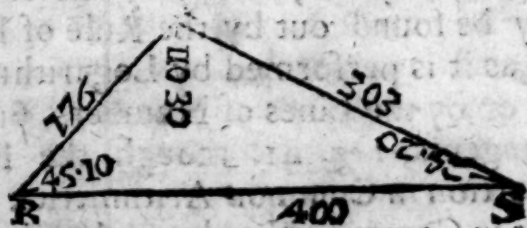
absolutely necessary, which for its excellency deserves to be writ in Letters of Gold, which together with the Golden Rule or Rule of Three, as it is wrought and performed by Logarithmes, pag. 31, and by the Lines of Numbers, Sines and Tangents, pag. 41, is sufficiently able to do whatsoever is necessary in Trigonometry, which take as follows.

In all Triangles whatsoever, every side is proportional to its opposite Angle, and every Angle to its opposite side: And further, as the Angle opposite to one side is to the Angle opposite to the other side, so is the sides themselves to one another, & *è contra*, the sides are to the Angles.

As for Example in the Triangle, pag. 134. As the Angle A 30 d. is to the perpendicular 231, so is the Angle C 60 d. to the Base 400, and so is the Radius or the right Angle 90 d. to the Hypothenuse 462, & *è contra*, so that there needs no more but having any Three of the Six parts of a Triangle given, any one, or every one of the rest may be found out by the Rule of Proportion, as it is performed by Logarithmes, pag. 31. or by the Lines of Numbers, Sines, and Tangents, pag. 41. though the Rule of Proportion in Common Arithmetick will not do it in Common Numbers, that is, will not find Angles. How to find Sides by Common

mon Arithmetick is taught before, pag. 136. But by Logarithmes or the Lines you may find not only Sides but Angles also: Taking this observation along with you, That when you meet with an Obtruse Angle, to find its opposite side, you must work by its complement to 180, which being used instead of the Angle it self, will effect the same thing in every respect. As to give an example in the Obtuse Angled Triangle QRS, whose Angle at Q is obtuse, being 110 d. 30 m. the Angle R 45 d. 10 m. the Angle S. 24 d. 20 m. the side RS opposite to the obtuse Angle is 400, the side QS 303 opposite to the Angle R; The side QR 176 opposite to the Angle S, so the proportions stand thus,

As 24 d. 20 m. the Angle at S is to 176, so is 69 d. 30 m. (the complement of 110 d. 30 m. to 180) to 400: And so is 45 d. 10 m. to 303, & *è contra*: Therefore say by the Rule of proportion, either by Logarithmes, or by Lines:



If 24 d. 20 m. give 176, what gives 69 d. 30 m. or 45 d. 10 m. to find the sides. Or,

If

If 176 give 24 d. 20 m. what gives 400, or 303 to find the Angles : see pag. 31 and 41, where you have the way of working these, or any such like proportions by the *Rule of Three*, either in Logarithmes, or by the Lines.

C H A P. XV.

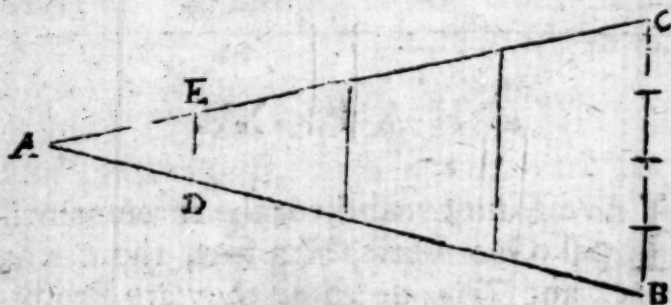
HAVING now handled the three principal Geometrical Figures, the *Circle*, *Square*, and *Triangle*; how they are Produced, Reduced, and Measured themselves; and consequently all other Figures or Bodies by their Rules; to conclude, What I have to say in reference to Geometry in general, shall be to shew you the several excellent uses of Mr. *Gunters Sector*, which being properly Geometrical, I reserved for this place, to shut up general Geometry.

S E C T. I.

First then, to shew the Ground of the Sector.

Let AB, AC represent the Legs of the Sector : Then seeing these two, AB, AC are equal, and their Sections AD, AE also equal, they shall be cut proportionably, and
if

if we draw the Lines BC, DE, they will be parallel by the Second *Prob. 6.* Book of *Euclid*, and so the Triangles ABC, ADE shall be equiangled, by reason of the Common Angle at A, and the equal Angles at the Base, and therefore shall have the sides pro-



portional about those equal Angles, by the fourth *Prob. lib. 6. Euclid.* Again the side AD shall be to the side AB as the Base DE unto the parallel Base BC, and by Conversion AB shall be unto AD, as BC unto DE. And by permutation AD shall be unto DE, as AB to BC, &c. so that if AD be the fourth part of the side AB, then DE shall be the fourth part of his parallel Basis BC, the like reason holdeth in all other Sections. And by the way, to understand the general aim and use of the *Sector*, it is to perform the same by Lines, which Arithmetick does by Numbers, as I shall shew you particularly: Now these lines which are found out by the Sector, are of two sorts: Either first
Lateral,

Lateral (or such as are found upon the sides of the Sector) as are the Lines, AB, AC : Or Secondly, Parallel such as are the Lines CB, DE from one side of the Sector to the other in its parallel points.

Having premised thus much in general, I shall shew you in particular the several uses of the Sector.

Now you must understand that (besides the Lines of Numbers, Sines, and Tangents upon the outward edge of the Sector, and a foot divided into 12 Inches ; as also another Line of equal parts, or a foot divided into 100, or to be supposed 1000 parts upon the Inner edge of the Sector.) There are these several Lines upon the Sector it self.

These four, as also the 2 last are the principal Lines.

1. A Line of Sines.
2. A Line of Lines.
3. A Line of Superficies.
4. A Line of Solids.

These 5 Lines are less principal, and for particular Uses.

5. The Line of Quadrature.
6. The Line of Segments.
7. The Line of inscribed Bodies.
8. The Line of Equated Bodies.
9. The Line of Metals.

The other two Principal Lines

10. The Meridian Line.
- Lastly, The Tangent Line.

And of these in the same order I here Rank or Reckon them, and first of the first (to wit) The Line of Sines.

S E C T. 2.

The Line of Sines.

1. To protract Angles by a Line of Right Sines is taught *pag.* 87, 88. And to avoid Tautology, I shall not here repeat it, but refer the Reader thereunto, being one excellent use of this Line.

2. If the Radius of a Circle be the same with the Lateral Radius or Semiradius of the Sector, the respective several Sines are the same as in this Line of Sines: But if it be either greater or lesser, set over the given Radius in the Line of Sines betwixt 90 and 90, and the Parallel Sines all along are the right Sines to the same Radius; and hereby you may divide any Line given, as a Line of Sines; and by the same Rule any sine being given, you may find the Radius thereof, by making it a Parallel Sine in its respective Number, the Radius will be betwixt 90 and 90.

3. To find the Chord of any Ark, observe that the Right Sine of the same Ark is half the Chord required; as the Sine of 10 is the Chord of 20, the Sine of 20 is the Chord of 40, &c. all along. The Radius or 60 degrees of a Line of Chords being set over as a Parallel betwixt 30 and 30
of

BOOK II. *Of the Sector.*

147

of the Line of Sines upon the Sector, mark this well.

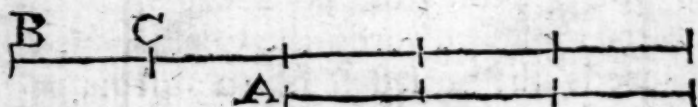
4. To find the versed Sine of any Ark. If you reckon from 90 at the end of the Sector to 80 towards the Center, that distance is the versed Sine 10; from 90 of to 70 the versed sine of 20, and so of the rest, if the versed Sine required exceed a Quadrant, as suppose 130 degrees, First, Take the whole Radius, and as many degrees as it is above 90, (as in this Example it is 40) add thereunto from the end of the Sector towards the Center.

S E C T. 3.

Of the Line of Lines.

This serves for a Line of equal parts upon any occasion, as they are numbred on the sides of the Sector, if the Radius be the same with the Lateral Radius. But if you would divide any other Line given into equal parts, set it over as a Parallel Radius betwixt 100 and 100, and the Parallel distances shall divide the same given Line into the same equal parts, according to the number which the points of your Compasses do stay upon: As if you would take or mark out 40 of those parts, the parallel distance betwixt 40 and 40 marked out upon the Line given is 40 of the said parts, and so of the rest.

Or, Suppose the Line B were to be divided into 5 parts, First, I take the Line B betwixt the Compasses, and to it open the



Sector in the point 5 and 5, so the parallel between the point 1 and 1 doth give me the Line BC, which doth divide the same into the parts required.

P R O B L. I.

To increase or Diminish a Line in a given Proportion.

As let A be a Line given to be increased in the proportion, as 3 to 5. First, I take the line A in the Compasses, and open the Sector till the points of the Compasses do reach just between 3 and 3 on the sides of the Sector; so the Parallel between the points 5 and 5 doth give me the Line B, which was required. In like manner, if B were to be diminished as 5 to 3, First, set over B betwixt 5 and 5, and its parallel betwixt 3 and 3 gives you A diminished as 5 to 3.

PROBL.

P R O B L. 2.

To find a proportion betwixt two or more right Lines given.

Take the greater Line given, and according to it open the Sector in the Points of 100 and 100; then take the lesser Lines severally, and carry them parallel to the greater till they stay in like points; so the number of Points wherein they stay shall shew their proportion to 100, so the Line B is in proportion to A, as 100 to 60, and A to B, as 60 to 100.

P R O B L. 3.

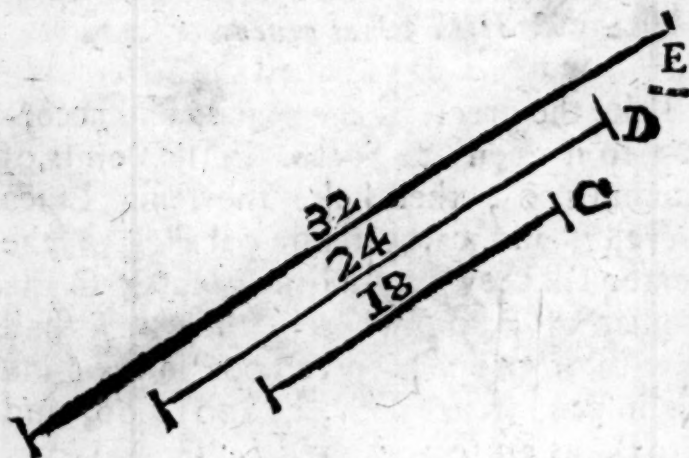
Two Lines given, to find a Third in continual Proportion.

Take the two Lines or Numbers given, and set them upon both sides of the Line of Lines (or equal Parts) from the Center, and Mark the Terms to which either of them extendeth; as suppose the Line C 18, and D 24 were the Lines or Numbers given. Measure with your Compasses the second Line D 24, and open the Sector at that distance in the Terms of the first Line or Number C, and so keep the Sector at this

H 3

Angle

Angle. The Parallel distance between the Terms of the second Lateral Number or



Line (24) being measured in the same Scale from whence his Parallel was taken, will give the third number proportional, *viz.* the Line E, containing 32 of the same equal parts.

P R O B L. 4.

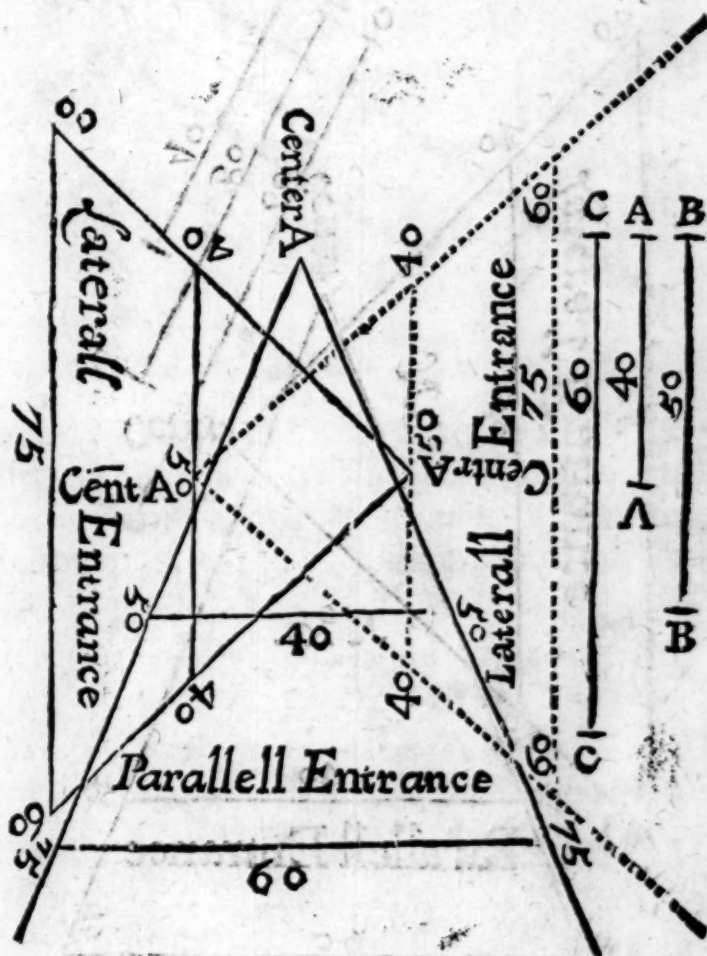
Three Lines or Numbers being given, to find a Fourth.

Let the Lines given be A 40, B 50, C 60, and the Question be this, If 40 Months give 50 £ . What shall 60 Months give?

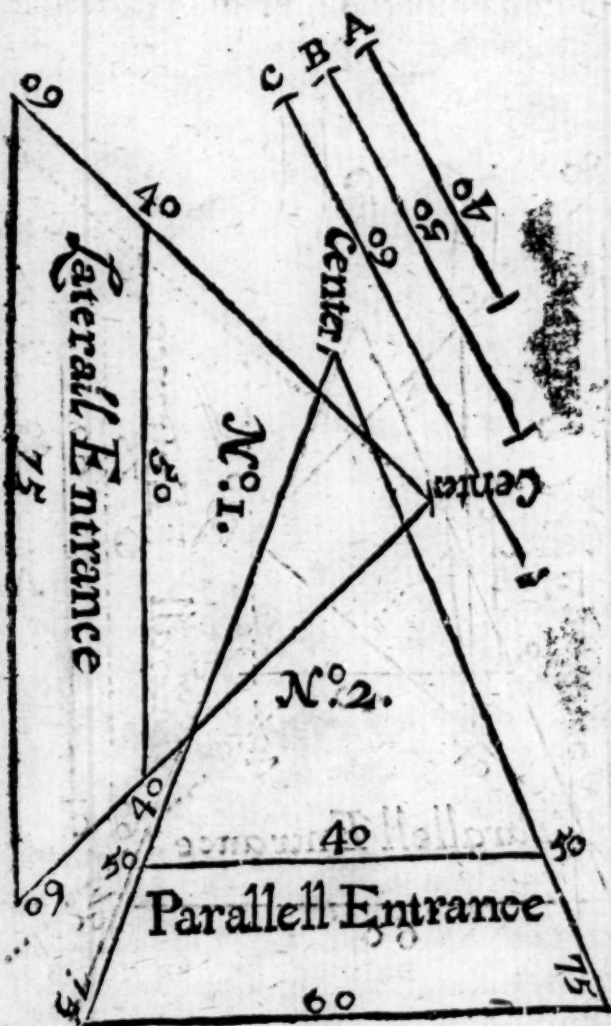
This you see is perfectly the Rule of 3, in

in the solution whereof a principal use of the *Sector* doth consist.

The first and third Lines or Numbers, (*viz.* A 40. C 60.) must be placed on both sides the *Sector* from the *Center*, and with the second, *viz.* B 50. open the *Sector* in the



Terms of the first, viz. A 40. And the Parallel distance between the terms of the third, viz. C. 60, shall be the fourth proportional, to wit 75, as in the Example; and this is called Lateral entrance.



But

But if you would enter with a Parallel, do thus. First, with the first Line or Number A 40 in your Compasses open the Sector to that distance in the Terms of the Second, viz. B 50 (or make it the distance between 50 and 50 on the sides of the Sector.) Then take with your Compasses the third Line or Number C 65, and with that distance observe where the Points of the Compasses do stay in the Parallels, and that is the fourth proportional, which you will find 75, as before.

S E C T. 4.

The Use of the Line of Lines, together with the Line of Sines, may be these.

P R O B L. 1.

In TRIGONOMETRY.

Any Two Sides and an Angle included given, to find the Third Side.

Let the Sector be opened to the Angle given (30 d.) and observing the length of either Side upon the Sides of the Sector in the Line of Lines, the parallel distance from one point to the other, where the Sides of the Triangle terminate, doth give the Side

H 5

requi-

required. As suppose the Base of the Triangle pag. 134, be one side given, viz. 400, the Hypothenuse the other side 462, the Angle included 30 at A, you will find the distances with the Compasses from 400, to 462, to be 231 the side required.

P R O B L. 2.

Three sides being given, to find an Angle.

Let the two containing sides be laid on the Lines of the Sector from the Center one on one line, and the other on the other; and let the third side which is opposite to the Angle required be fitted over in the Terms of the other two; so shall the Sector be opened in those Lines to the Quantity of the Angle required. As for Example in the before-mentioned Triangle, pag. 134.

Having

Having one of the containing sides 400 upon one side of the Sector, and 462 on the other, and setting these at the distance 231, the Angle at the Center will be found 30 *d.* Or suppose one side 72 in the Line of Lines, the other 54, and fit the opposite side (to the Angle required) over between 72 and 54; then with your Compasses measure the distance betwixt 50 and 50 in the same Line of Lines, and the same distance measured in the Line of Sines shall shew half the quantity of the Angle required.

As suppose the three sides given be AD 100, AC 75, AB 35. I might take 35 for the side CD out of the Line of Lines, and set it over from 100 to 75. This done I take the distance between 50 and 50, and measuring it in the Line of Sines, I find it to be 8 *d.* 8 *m.* the double whereof is 16 *d.* 16 *m.* the Angle required.

S E C T. 2.

The Use of the Line of Lines, together with the Line of Sines in Navigation, see in its proper place, That is,

1. To know how many Leagues do answer,

swer to a Degree of Longitude in every several Latitude.

2. To know how many Leagues do answer to a Degree of Latitude in every several Rhumb. *Vide ut supra.*

S E C T. 5.

The Use of the Line of Lines, together with the Line of Superficies in Extracting the Square Root.

1. The Line of Superficies is said to be divided into 100 parts; therefore hath upon it twice the Figure 1, the former signifying Units, the latter Tens in that respect of the Line being divided into 100 parts.

2. Observe, if the Number, whose Square Root you desire, consist of even Figures, as 2, 4, 6, 8, Figures, or the like, the Number given shall fall out between the tenth division and the end of the Sector, (or which is all one) between the latter Figure of 1, and the end of the Sector: But if the Number (whose Square Root you desire) be odd, that is 1, 3, 5, 7, Figures, or the like, the Number given shall fall (or must be measured) between the Center of the Sector, and the Tenth Division.

3. These

3. These being observed, set one foot of your Compasses in the Center, extend the other to the Term of the Number given in one of the Lines of Superficies, (which suppose 36.) The same distance applyed to one of the Line of Lines, shall find 6 for the Square Root.

Likewise to Square a Number given, first Extend the Compasses upon the Line of Lines to the Number you would Square (as suppose 6,) and apply the same distance to the Line of Superficies, will give (36) as before.

S E C T. 6.

*By the Line of Lines, and Solids, to find
the Cube Root.*

In finding a Cube Root of a Number given, prick the first Figure, and after that every third Figure from the right hand to the left; and if the last prick to the left hand fall under the last Figure, the Number given shall be reckoned at the beginning of the Line of Solids, from the Center to the first 1, and the first Figure of the Root shall be always either 1 or 2.

If the last prick fall under the last Figure but One, Then the Number given shall be reckoned from the Center to the
second

second 1, and the first Figure of the Root shall be always either 2, 3, or 4.

But if the last prick fall under the last Figure but two, then the Number given shall be reckoned from the Center between the last Figure of one, and the end of the Sector : For you must know, as the Line of Superficies is supposed to be divided into 100 parts, and so having 2 Figures of 1 upon it, the first signifying Units, the second Tens ; so the Line of Solids is supposed to be divided into 1000 parts, and hath three Figures of one upon it, the first signifying Units, the second Tens, and the Third Hundreds ; so that according to this Division the whole Line is 1000 as before. These things being observed, the Rule for finding the Cube Root is this.

Set one Foot of your Compasses in the Center of the Sector ; Extend the other in the Line of Solids to the point of the Number given according to the Rules before given : This distance applied to the Line of Lines, shall shew the Cube Root of that Number.

Or to Cube a Number, First, Extend the Compasses upon the Line of Lines and then measure the same distance upon the Line of Solids, it will give you the Cube Number of a Cube Root given.

If the Reader desire further concerning these two last propositions of finding the Square and Cube Root by the Sector, I refer him to *Gunter* himself upon this Subject.

S E C T. 7.

Of the Line of Superficies alone or single: As also the Line of Solids alone or single.

Observe first, That the Line of Superficies and Solids do serve to find Proportions betwixt Square and Cubique Bodies: As the Line of equal parts do betwixt Lines (from thence called properly the Line of Lines) therefore the same Rules may in a manner serve for these which are given for the other right applyed, *mutatis mutandis*.

Secondly observe, that whatsoever is said concerning Superficies, the very same Rules serve for Solids, only that they are wrought upon different Lines. Therefore to avoid Repetition, observe to apply these Rules given for Superficies to Solids also, *mutatis mutandis* as I said before.

P R O B L.

P R O B L E M I.

Therefore to proceed to find a Proportion between two or more like Superficies (as is before done in Lines,) do thus,

Take one of the sides of the greater Superficies, and according to the said side given open the Sector in the points of 100 and 100, then take the like side of the lesser Superficies, and carry it parallel to the former, till they stay in like Points, so the Number of Points wherein they stay shall shew the proportion to 100 in the Lines of Superficies: As for Example, Let A and B be the sides of like Superficies. First, I take the side A in the Compasses, and to that distance open the Sector in the points of 100 and 100: Then keeping the Sector to that Angle, I enter the lesser side B parallel to the former, and find it to cross the Lines of Superficies in the point 40 and 40, therefore the Proportion of the Superficies, whose side is A, to that whose side is B, is as 100 to 40, or in lesser Numbers, as 5 to 2. This Proposition may also be wrought by 60, or any other Number that admits several Divisions.

It may also be wrought without opening the Sector, for if the sides of the Superficies

cies given be applied to the Lines of Superficies, beginning always at the Center of the Sector, there will be such proportion found between them, as between the Number of parts whereon they fall.

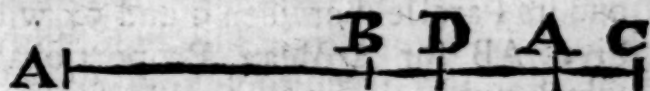
What is said of Superficies, the like is to be understood of Solids wrought upon their own Lines.

P R O B L. 2.

To Augment or Diminish a Superficies in a given proportion; or (which is the same) to add one Superficies to another, or subtract one from another, will be all understood by this Example following.

Suppose A and B were the sides of two Squares, or the Diameters of 2 Circles, and it were required to make a third Square or Circle equal to them both.

First, The proportion between A and B will be found to be as 100 to 40, or in lesser numbers, as 5 to 2; then because 5 and 2 added together make 7, I augment the side A in proportion of 5 to 7, and produce the side C, on which if I make a



Square, or make it the Diameter of a Circle, the said Square or Circle shall be equal to both the Squares of A and B, or to two Circles

Circles made upon their Diameters, which was the thing required. To subtract one from another is after the same manner, as from 5 take 2, the Remainder is 3, so it must be diminished in proportion of 5 to 3, which is D.

P R O B L. 3.

To find a Mean proportional between 2 Lines, which may be supposed the sides of a Square, or Diameters of a Circle, and thereby to find several Corollaries.

Suppose the Lines given be AA and AC. First find the proportion betwixt them as they are Lines as is taught pag. 60. which will be found to be as 4 to 9, wherefore I take the Line AC, and put it over to the Lines of Superficies between 9 and 9, and keeping the Sector at this Angle, his Parallel between 4 and 4, gives me AB for the mean Proportional: Or take the Line AA, and put it over in the Lines of Superficies between 4 and 4, and its Parallel between 9 and 9, will give AB for a Mean Proportional, which upon the same Scale of equal Parts will be found 6; for as 4 is to 6, so is 6 to 9.

Upon

Upon finding out this Mean Proportional depend many Corollaries, especially these following.

First to make a Square equal to any Superficies given.

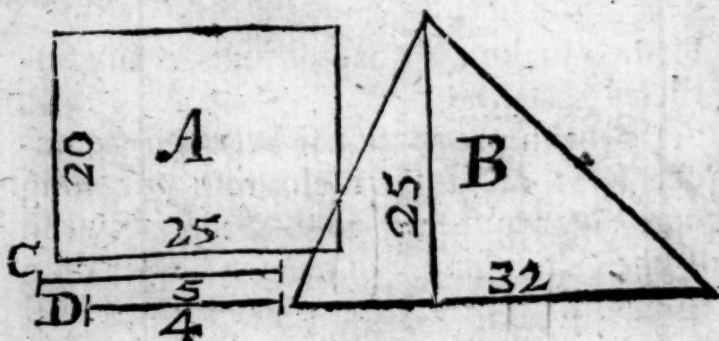
As a mean proportional between the unequal sides of a Parallelogram (or long Square :) or a mean proportional between half the Base, and the perpendicular of a Triangle shall be the side of a Geometrical Square equal to either of these in superficial Content. And if it be any other right-Lin'd Figure, it may be resolved into Triangles, and so a side of a Square found equal to every Triangle, and these reduced to one equal Square, it shall be equal to the whole Right-Lin'd Figure given.

Secondly, To find a Proportion between several Superficies though unlike to one another.

As for Example, The Oblong A, and the Triangle B are Superficies unlike to one another, therefore to find a Proportion between these.

First, Between the sides of A, viz. 20, and 25, I find a mean proportional, as C. This is the side of a Square equal to A, then between the perpendicular of the Triangle B, and half his Base, I find a mean proportional,

portional, viz. D. This is the side of a Square equal to B.



But the proportion between the Squares of C and D will by the Rule for it pag. 160 be found to be as 5 is to 4, *vide* the Rule it self.

Thirdly, to make a Superficies like to one Superficies, and equal to another.

Let one be the Triangle A, the other the Rhomboides B, First, between the Perpendicular and the Base of B, I find a mean proportional, and note it in B, as the side of a Square equal to B the Rhomboides. Then between the perpendicular, and half the Base of the Triangle A find a mean proportional, and note it in A as the side of his equal Square.

Wherefore as the squared side of B is to the squared side of A, so shall the longer side.

ving the sides and perpendicular, you may frame the Rhomboides up, and it will be equal to the Triangle A, which was the thing required.

And because here is mention made of protracting or framing up a Rhomboides, having the longer side, shorter side, and perpendicular given, I shall here shew you how to do it.

First then, Draw the length of the longer side, or the Line AB in the former Example. And then taking the Length of the shorter side in your Compasses, setting one foot in B, draw an Arch of a Circle *a b*, as also setting one foot of your Compasses in A, draw the Arch *c d*. Then take the length of the perpendicular in your Compasses, and setting one foot in B, again draw the Arch *e f*. and removing to A, draw the other Arch *g h*. These 4 Arches being drawn, lay your Ruler so as just to touch the Arches *e f* and *g h*, and draw a parallel Line from the point D, where the Ruler crosses the Arch *c d*, to the point E, where the Ruler will cross the Arch *a b*; and lastly from the point D draw the Line DA, and from the point E draw the Line EB, and let fall the perpendicular AC from the point A, so will you have a Rhomboides as was required.

S E C T 8.

Of the Lines of Quadrature.

The Lines of Quadrature may be known upon the Sector by the Letter Q, and by their place between the Lines of Sines.

Q signifies the side of a Square, 5 the side of a Pentagon, with 5 equal sides, 6 of an Hexagon, that is, a Figure with 6 equal sides, and so 7 for the side of an Heptagon, 8 of an Octagon, 9 of a Nonagon, &c. S stands for the Semidiameter of a Circle, and 90 for a line equal to 90 degrees in the Circumference of the same Circle.

P R O B L. 1.

The use of these Lines may be to make a Square equal to a Circle given, or to make a Circle equal to a Square given.

If the Circle be given, take his Semidiameter, and to it open the Sector in the points at S; so the parallel taken from between the points at Q, shall be the side of the Square required: or if the Square be given, Take his side, and to it open the Sector in the points at Q; so the Parallel taken from between the points at S, shall be the Semidiameter of the Circle required.

P R O B L.

P R O B L. 2.

To Reduce a Circle given, or a Square into an equal Pentagon, or other like Sided, and like Angled Figures.

Take the side of the Figure given, and fit it over in his due points, so the parallel taken from between the points of the other Figures, shall be the sides of those Figures, which being made up with equal Angles, shall be all equal one to the other.

Other Superficial Figures not here set down may first be reduced into a Square, and then into a Circle, or other of these equal Figures as before.

P R O B L. 3.

In the third place, To find a Right Line equal to the circumference of a Circle, or other part thereof.

Take the Semidiameter of the Circle given, and to it (or to the same distance) open the Sector in the Points at S, so the parallel taken from between the points at 90, in this Line, shall be the fourth part of the circumference, &c.

S E C T. 9.

Of the Lines of Segments.

The Lines of Segments are here placed between the Lines of Sines and Superficies, and are numbred by 5, 6, 7, 8, 9, 10.

The use of them is to divide a Circle given into 2 Segments, according to a proportion given, or to find a proportion between a Circle and his Segments given: For the Numbers 5, 6, 7, &c. do represent the Diameter of a Circle so divided into 100 parts, as that a right line drawn through these parts perpendicular to the Diameter, shall cut the Circle into two Segments, of which the greater Segment shall have that proportion to the whole Circle, as the Parts cut have to 100: For Example, Let the Sector be opened in the Points of 100 to the Diameter of the Circle given, as suppose BL, or any other Diameter; so a Parallel taken from the parts proportional to the greater Segment required, shall give the depth of that greater segment: As if the Diameter of a Circle were BL, carry the greater segment LO when the Sector is opened in 100 and it will stay in 75, which shews the proportion



portion is as 75 to 100, or that the segment to the Circle is 3 parts of 4. Hereby you may find the side of a Square equal to any known segment of a Circle: For as the proportion is of the Segment to a Circle, so is the part of a Square to the Square equal to the whole Circle, which is taught before.

S E C T. 10.

Of the Lines of Inscribed Bodies.

These are placed between the Lines of Lines, and may be known by the Letters D, S, I, C, O, T, of which D signifies the side of a Dodecahedron, I of an Icosahedron, C of a Cube, O of an Octahedron, and T of a Tetrahedron, all inscribed into the same Sphere, whose semidiameter is here signified by the Letter S.

The Uses are

P R O B L. 1.

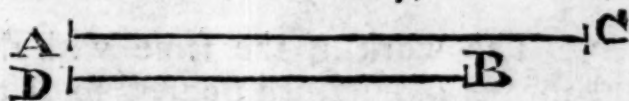
The Semidiameter of a Sphere being given, to find the sides of the 5 Platonick or Regular Bodies, which may be inscribed in the same Sphere.

P R O B L.

P R O B L. 2.

The side of any of the five Regular Bodies being given, to find the Semidiameter of a Sphere that will circumscribe the said Bodies.

If the Sphere be first given, take his Semidiameter, and to it open the Sector in the points at S, and the Parallel distance betwixt any other of the Bodies will give you the side of the same Body, which may be



inscribed in the Sphere: As if the Semidiameter be AC, the side of a Dodecahedron, to be therein inscribed will be DE.

If the side of any other Body be first given, fit it over in his due points, and his parallel distance between the points S will give the semidiameter of a Sphere which shall circumscribe the same Body.

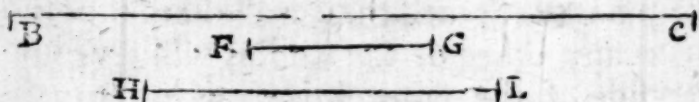
S E C T. II.

Of the Lines of Equated Bodies.

These are placed between the Lines of Lines and Lines of Solids, noted as the former with these Letters D.I.C.S.O.T, signifying the same things.

The Use is this,

The Semidiameter of a Sphere given, to find the sides of the five Regular Platonick Bodies equal to that Sphere: Or the sides of any of the five Bodies given, to find the Semidiameter of a Sphere, and the sides of the other Bodies equal to the first Body



given. The work is the same with that of the former of inscribed Bodies, so if the semidiameter of a Sphere be BC, the side of a Dodecahedron equal to this Sphere will be found FG, and of an Icosahedron HL, and so of all the rest.

S E C T. 12.

Of the Lines of Metals.

The Lines of Metals are here joyned with those of Æquated Bodies, and are noted with these Characters, ☉ ♄ ♅ ♆ ♇ ♈, of which ☉ *Sol* stands for God, ♄ *Mercury* for Quicksilver, ♅ *Saturn* for Lead, ♆ *Luna* for Silver, ♇ *Venus* for Copper, ♈ *Mars* for Iron, and ♉ *Jupiter* Tinn.

The

The use of them is to give a Proportion between these several Metals in their magnitudes and weight, according to the Experiments of *Marinus Ghetaldus* in his Book called *Promotus Archimedes*.

P R O B L. 1.

In like Bodies of several Metals, and equal weight; having the Magnitude of the one, to find the Magnitude of the rest.

Take the Magnitudes given out of the Lines of Solids, and to it open the *Sector* in the points belonging to the Metal given; so the Parallel taken from between the points of the other Metals, and measur'd in the Lines of Solids, shall give the Magnitudes of their Bodies.

P R O B L. 2.

In like Bodies of several Metals and equal Magnitude, having the weight of the one, to find the weight of the rest.

As if a Cube of Gold weighed 38 pounds, and it were required to know the weight of a Cube of Lead, having equal Magnitude: First, I take 38 for the weight of the Golden Cube out of the Lines of Solids,

I 3

and

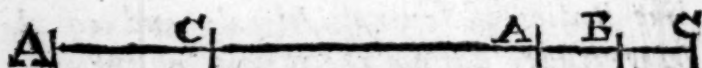
and put it over in the points of ♄ *Saturn* belonging to Lead; so the Parallel distance taken from between the points of ☉ *Sol* standing for Gold, and measured in the Line of Solids doth give the weight of the Leaden Cube required, to be $13\frac{1}{2}$.

P R O B L. 3.

A Body being given of one Metal, to make another like unto it of another Metal, and equal weight.

Let the Body given be a Sphere of Lead, containing in Magnitude $16d.$ whose diameter is AA , to which I am to make a Sphere of Iron of equal weight.

If I take out the Diameter AA , and put it over in the points of ♄ *Saturn* belonging to Lead, The Parallels taken from between the joints of ♂ *Mars* standing for



Iron shall be AB the Diameter of the Iron Sphere required, and this compared with the other Diameter AA in the Line of Solids, will be found to be $23d.$ in Magnitude; C is the length of the whole line, and from A to C again.

P R O B L.

P R O B L. 4.

A Body being given of one Metal, to make another like unto it of another Metal, according to a weight given.

As if the Body given were a Sphere of Lead, (or a Round Lead Bullet) whose Diameter is AA, and it were required to find the Diameter (of a Sphere of Iron) or of an Iron Bullet, which Iron Sphere shall weigh three times as much as the leaden Sphere or Bullet. I take AA and put it over in the points of ι which stands for Lead: His Parallel taken from between the points of δ signifying Iron, shall give me AB for the Diameter of an equal Sphere of Iron: If this be augmented in such proportion as 1 to 3, it gives C for the Diameter required of a Sphere three times the weight of the former.

Having largely discoursed of *Geometry* in General, and of the three Famous Geometrical Figures, the *Circle*, *Square*, and *Triangle*, and having shewed you the Making and Measuring of these, and the Reducing of all other Figures, whether Regular, or Irregular, to some of these, if not to the two former, yet at least to the last of these,

viz. a *Triangle*, and by consequence to Measure the same, by the same Rules that Measure a *Triangle* (or *Trigonometry*); Adding in conclusion the use of the *Sector*: I come now to Treat of *Geometry* according to the Proper and Genuine Etymology of the Word, which is derived from γῆ Terra & μέτρον mensura, that is, *The Measuring of the Earth*, and that (as elsewhere explained) to be understood the whole Globe of the Earth, containing *Sea* and *Land*: That which concerns the *Land* is called *Surveying*, and that which concerns the *Sea*, *Navigation*: And of these in particular. But first of Measuring this whole Globe of *Sea* and *Land* together in the General.

OF THE
GLOBE
OF THE
EARTH.

CHAP. I.

To Measure the whole Globe of the Earth comes under the Rules of the first Geometrical Figure, to wit, the Circle, and that a Solid Circle, (taking it for granted, That the Earth and Sea together make one Globular or Spherical Body.)

NOW we must find the solid Content of
a Globe by the superficial Content of
15 a Globe,

a Globe, and the Superficial Content of a Globe by the superficial Content of a Circle, having the like Diameter. This Diameter, we must find by the circumference of the same Circle: And lastly, This Circumference (as all other Circumferences whatsoever) must contain 360 degrees. Now if we would take the Circumferences of the Earth both Sea and Land in *English* Miles, we must know how many of these Miles upon the Earth are answerable to a Degree in the Heavens.

In the just Quantity and Measure of a Degree, the Common Rule is much out, which accounts 5 Foot to a Pace, and 1000 paces, *i. e.* 5000 feet to a Mile, and 60 such Miles, or 300000 Feet to a Degree.

The *English* Mile, which by the Statute is somewhat more, then this will not serve the turn, which is thus, 16 $\frac{1}{2}$ Feet make a Pole, 40 Poles a Furlong, 8 Furlongs a Mile, 60 such Miles contain 316800 Feet: By this account 1760 Yards is a Mile.

But Mr. *Norwood's* Experiment by knowing the difference of Elevation of the Pole betwixt *York* and *London*, and measuring exactly the way betwixt them, passes for currant with all the Mathematicians I meet with of late, which was made on this manner. *June 11. 1635, At York*, by an Arch of a Sextan of more than 5 Foot semidiameter,

meter, He took the Meridian Altitude of the Sun which was 59 Degrees, 33 Minutes, Latitude 53 d. 58 m. June 11. 1633, in *London* he did the same, and found the Meridian Altitude of the Sun to be 62 Degrees, 1 Minute; in Latitude 51. 32 : Now deducting the less latitude from the greater, there remains 2 Degrees, 28 Minutes, which is the difference of Latitude, (*Tork* being in 53 Degrees 58 Minutes, and *London* in 51 Degrees 32 Minutes) as above noted : So by this was found 69 English or Statute Miles; 4 Furlongs, 14 Poles, 9 Feet in a degree; or 367200 Feet. But to make 60 *English* Miles in a Degree, each Mile ought to contain 6120 Feet : These multiplied by 60, make 367200 as above, and those again by 360 is 13219200 Feet for the circumference of the whole Earth.

Now according to this observation of Mr. *Norwoods*, there will be according to statute Measure 69 Miles, 4 Furlongs, 14 Poles and 9 Feet *English* in every Degree. Yet that we may not seem to differ so much from former Writers, and the Common Account we shall still reckon only 60 *English* Miles to a Degree, and allow so many more feet to a Mile, viz. 6120 Feet, and 2040 yards; and if we cannot allow them the name of *English* Miles, we may call them *Mathematical* Miles, (though no question

question but our *Yorkshire* Miles may allow of the same Length.) Therefore,

First, 60 Miles allowed for a Degree, that multiplied by 360, makes 21600 Miles for the whole circumference of the Earth.

Secondly, And by this Circumference to find the Diameter, Multiply 21600 by 7, is 151200, and that divided by 22, makes $6872 \frac{1}{2}$ for the whole Diameter (according to the Rule, pag. 78.) the semidiameter (rejecting the Fraction) is 3436 Miles to the Center of the Earth.

Thirdly, Now having the Diameter, to find the Area or Superficial Content of a Circle having the like Diameter: (by the third Rule, pag. 78.) Multiply the Diameter by it self, that is 6872 (omitting the Fraction) by 6872, it produceth 47194384 Miles; this Multiplied again by 11 makes 519138224, and divided by 14, the Area of the Circle is 3708130 Miles.

Fourthly, This superficial Content of the Circle multiplied by 4, (by the Rule pag. 80.) produceth for the superficial Content of the Globe 148325204 Miles. And,

Fifthly, For the solid Content, Multiply the superficial Content by the sixth part of the Diameter, viz. $1145 \frac{1}{3}$, and it brings forth 1698816003 $1 \frac{2}{3}$, which are so many solid Miles; or the solid Content in Miles
(of

(of 6120 feet in a Mile) of the whole Terrestrial Globe, containing *Sea* and *Land*. And so much concerning the Measuring of the Earth in general: It follows that we treat of *Surveying* and *Navigation* in particular: Only by the way take these two Corollaries.

1. Supposing the aforesaid Diameter of the Earth 6872 $\frac{1}{2}$ Miles. If you imagine a hole through the Earth, and that a Milstone should be let fall down into this hole, and to move a Mile in a Minute of time, it would be more than two days and a half before it would come to the Center, and being there, would go no further, but as it were, hang in the Air.

2. Supposing the Circumference 21600 Miles, if a Man should go every day 20 Miles, it would be three years wanting but a Fortnight before he could go once about the Earth.

And if a Bird should fly round about it in two days, then must the motion be 450 Miles in an hour.

OF SURVEYING.

CHAP. II.

Surveying in a strict and limited sense may be taken only for the Measuring of some Piece or Parcel of Land : But in a larger sense, I take it here for the taking of all manner of Heights and Distances at Land, whether the same be Accessible, or Inaccessible.

TAking of Heights being a Resolving of a Triangle, standing upwards or vertically upon his Horizontal Base, and taken by a Quadrant with Thread and Plummet : Taking of Distances is a resolving of a Triangle (supposed to lye Horizontally Flat,) and taken by a Theodolite or such like Instrument, which is no other but a Quadrant laid Flat with

with an Index, and Sights instead of a Thread and Plummer: Not to exclude other Instruments, as the *Cross-Staff*; for taking both Heights and Distances both at Sea and Land; but especially at Sea: As also the *Plain Table*, an Instrument very much admired by many, of which you shall hear more hereafter.

As likewise I shall say something of the Circumferentor, another Useful Instrument for Surveying; but that which I do most admire and use is the Quadrant for Heights, and the Theodolite for Distances, which is no more but a Circle, a Semicircle, or Quadrant laid Flat, divided into Degrees and Minutes with an Index and Sights, as any other Quadrant (properly so called) is held upright, and used with a Thread and Plummer.

Both these (that is) both the Quadrant and Theodolite may be notably supplied by *Gunters Sector*, being set at the distance of a Quadrant (or 90 degrees) and having a loose piece fastned with two Mortices, and divided into Degrees for the quadrantal side, having also a Thread and Plummer fastned in the Center, when you use it one way for a Quadrant, and a Pin for an Index and Sights to turn upon when you use it in the other way for a Theodolite, being usually each Leg just a foot in length
from

from the Center; the Quadrantal or loose piece may be first divided into 90 degrees, and every degree into 6 parts, so that every one of the lesser Divisions are 10 Minutes, and these 10 may be supposed to be subdivided into other 10, which signifie single Minutes, which is as much as may be for an Instrument of no larger Radius. When it is used as a Theodolite, there must be screwed to it a socket of Brass, whereby the whole Instrument may turn upon the top of a three-leg'd staff (hooped also with Brass, as there shall be occasion) with a screw on the side of the socket to fasten the Instrument upon the staff in any position you desire: There may be also (to make it compleat) a Card and Needle screwed upon the Center, and the Index which carries the sights, being twice the Length of the Legs, (that is) 2 foot long, another of the same length without sights may cross it at Right Angles; so that making a perfect cross, moving upon the Center as one part goes off the Quadrant, another comes on, which is as good as if it were a whole Circle. This may suffice for the description of the Quadrant and Theodolite, I shall now shew you how to take Heights and Angles of Altitude by the Quadrant, and to take Distances or Angles of Position by the Theodolite: And to take both Heights and

and Distances by the Cross-staff, whether the same be Accessible, or Inaccessible.

S E C T. I.

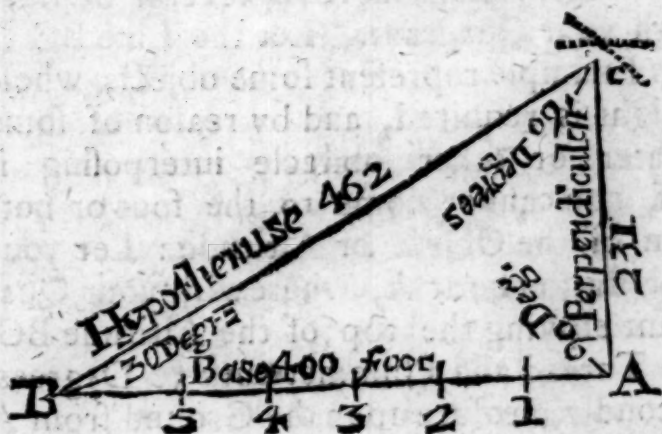
To take any Height or Altitude, whose Foot or Basis is accessible at one station.

This is no more but the resolving of a plain Right Angled Triangle. As suppose the perpendicular CA were a Tree, Tower, or Steeple, or any other thing whose Height is required. First, standing at any convenient distance from the foot of the Object to be measured as at B, and there looking through both the sights of your Quadrant till you espy the top of the Altitude at C, observe what Degrees and Minutes are cut on the Limb of the Quadrant by the Thread, which suppose 30 degrees, as in the Example: These 30 degrees are the quantity of the Angle CBA, (or more plainly to call it the Angle B.) Now because that every right lin'd Triangle contains 180 degrees of a Circle, and every right Angle contains 90 of them; the Angle A being a right Angle, the other two Angles, that is B and C must contain other 90, whereof B by the Quadrant appears to be 30: Therefore C must needs be 60 degrees, the complement of 30 to 90. And lastly,
You

You must measure by a Yard, Chain, or a Two foot Rule, the Base of the Triangle, that is, the space or distance betwixt B, the place of your standing, and A the foot or bottom of the Altitude (which suppose to be 400 foot) As in the Example.

Now these three parts of the Triangle being given or known, that is two Angles and a side, *viz.* the Angle B 30 degrees, and by consequence the Angle C 60 degrees, and the length of the Base 400 Foot: You may by Trigonometry and the Rule of Proportion find any of the other three parts of the same Triangle, *viz.* the Angle A, the Hypothenuse BC, or (that which is here required) the Height or Length of the Perpendicular CA. You will find that any Angle or side, (and consequently this side the perpendicular here required) may be found several ways, but by these three ways especially: First, By Protraction, *pag.* 85, 86, 87. Secondly, By Arithmetical Operation, *pag.* 137, 138. But especially by that general Rule in Trigonometry, *pag.* 141, that every Angle is proportional to its opposite side, and every side to its opposite Angle: Therefore as the Angle C 60 Degrees is to the known side BA 400; so is the Angle B 30 Degrees to its opposite side sought; the Perpendicular CA, or the Height required, which is no more

more but performing the Rule of Three (or by these three Numbers given, to find a fourth;) either first by Logarithmes, as is taught *pag.* 41. Or, Secondly, By the Lines of Numbers, Sines and Tangents, *pag.* 41. Or lastly, by the Sector, *pag.* 150, 151. by placing the Numbers on this manner.

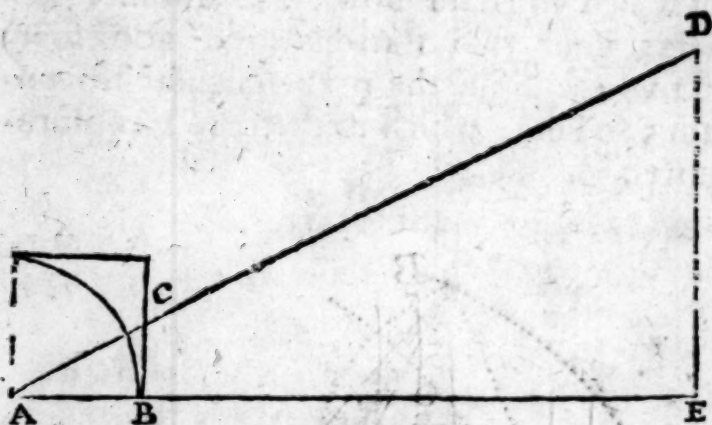


If 60 degrees give 400 Foot, what gives 30 degrees? The Answer by all the fore-mentioned ways will be found to be 231 *ferè* the length of the perpendicular, or the Height required: But to avoid Repetition, I shall refer the Reader to the Pages afore-mentioned for the manner of Operation.

S E C T. 2.

To take an Inaccessible Altitude at Two Stations.

For the effecting hereof, you must take two observations at two several Stations with your Quadrant. Let the Line BC in this Example represent some object, whose Height is required, and by reason of some Water or other obstacle interposing it self, you cannot come to the foot or bottom of the Object or Altitude: Let your first station be at A, wherewith your Quadrant espying the top of the Altitude BC, the Thread and Plummets cuts 50 Degrees. Secondly, measure upon the Ground from A to D 200 foot, and again standing at D, and espying the top of BC, observe what Degrees are cut by the Thread upon the Limb or Edge of your Quadrant, which suppose 64 *d*. Then fall to Protracting, and first draw an Angle at A 50 Degrees, and measuring 200 foot to D, protract there at D another Angle of 64 degrees, and from B (that is from the point where the two lines AB and BD do intersect or cut one another,) let fall a perpendicular, as is taught *pag. 55. Probl. 3.* and measure the Length of the said perpendicular by the same Scale
by



following Example,) with an Index and Sights; for as the Radius AB is to the Tangent BC, so is AE the distance to ED the Altitude.

S E C T. 3.

By the Height of the Sun, and length of the shadow, to find the Height of any Tree, House, Steeple, or the like.

First, Take the Suns Height by a Quadrant, which suppose 37 Degrees, and the length of the shadow of the said Tree, (which suppose 40 Feet) Then say as Radius to the length of the shadow of the Tree 40 Feet, so is the Tangent of the Suns height 37 Degrees to the height of the Tree desired, viz. $30 \frac{1}{8}$ feet.

Note that when the Sun is 45 degrees high

high, then the shadow is equal to the height; at 26 d. 56 m. the length of the shadow is double to the height; at 18 d. 43 m. it is 3 times; at 14 d. 4 m. it is 4 times the Height, and at 11 d. 31 m. the shadow is 5 times the Height of any Altitude. Or, you may know the Height of any thing by a walking-staff, or a Yard, or a two foot Rule by the shadow, thus.

Set the staff upright, and measure how often the shadow of the Staff is longer or shorter than the Staff, so in proportion is the shadow of the Tree to the height of the Tree it self. The like may be done by a Mafons Square set upright.

S E C T. 4.

To take the Height of any thing accessible by 2 straws or sticks laid cross one another.

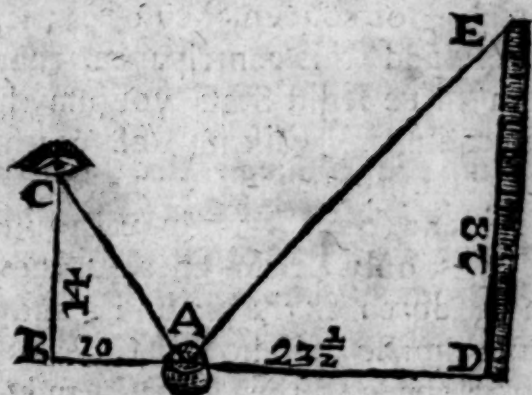
Take two straws or sticks of equal length, and fasten them perpendicularly cross one another just in the middle of both, and go backwards or forwards till the bottom and top of the Altitude be even with the ends of the Cross-stick, laying your Eye to the end of the other stick, and then is the height exactly as much as the distance measured to the Foot of the Altitude, adding to the Height found the Height of your Eye from the ground.

S E C T.

S E C T. 5.

To take an Altitude by a Bowl of Water, or an ordinary Looking-glass.

Place on the ground a Bowl of Water, which done, erect your Body strait up, and go back in a right line from the building or other Altitude, till you espy in the Center or Middle of the Water the very top of it; which done, observe the place of your standing, and measure the height of your Eye from the ground, together with the distance of your standing from the Water; and the distance of the Water from the Base or Foot of the Altitude: Then say as the distance from your standing to the water is to the Height of your Eye; so is the distance from the Water to the Foot of the Altitude, to the Height of the Altitude required. The same may be done by a Glass, if the Glass be placed Horizontal, or Flat upon the ground, and the Eye see the top of the Tower, &c. in the Angle or Edge of the Glass, and in the Line of Reflection: I shall give you only one Example for both: Suppose the Bowl of Water or the Glass be A, which laid flat upon the Ground, or Horizontal; go backwards till you espy the top of the Altitude E, then measure how many



many parts is from your standing at B to A, and also from B to your Eye at C, likewise the distance from A to D, and then say as the distance BA 10 is to the Height of your Eye from the Ground CB 14, so is the distance from A to D the foot of the Altitude $23 \frac{1}{2}$ to the Altitude required, which you will find 28 of the same parts.

CHAP. III.

THUS much may suffice for taking of Heights, in the next place we come to shew how to take Distances; and that (as I told you before) is done the best by the Theodolite, which is the most excellent

K

Instru-

Instrument for that use (especially as I have contrived it upon the Sector) and also the most portable for conveniency of carriage; I shall only add this contrivance, that instead of a three leg'd Staff you may have only an ordinary walking-staff with an Hoop instead of the Head for the Socket of the Instrument to move upon, and instead of three Legs, only a Brass cheep at the end to prick it down in the ground; and the Staff may be made with hollow Cases to put in the cross peece, the Index, and the Sights, and if you please, a pair of Compasses, whereby it becomes a general Instrument both for Observation and Calculation; the Sector you carry in your Pocket, and the Staff in your Hand, appearing no other but an ordinary walking Staff, which for neatness and portability exceed any other contrivance I meet with.

S E C T. I.

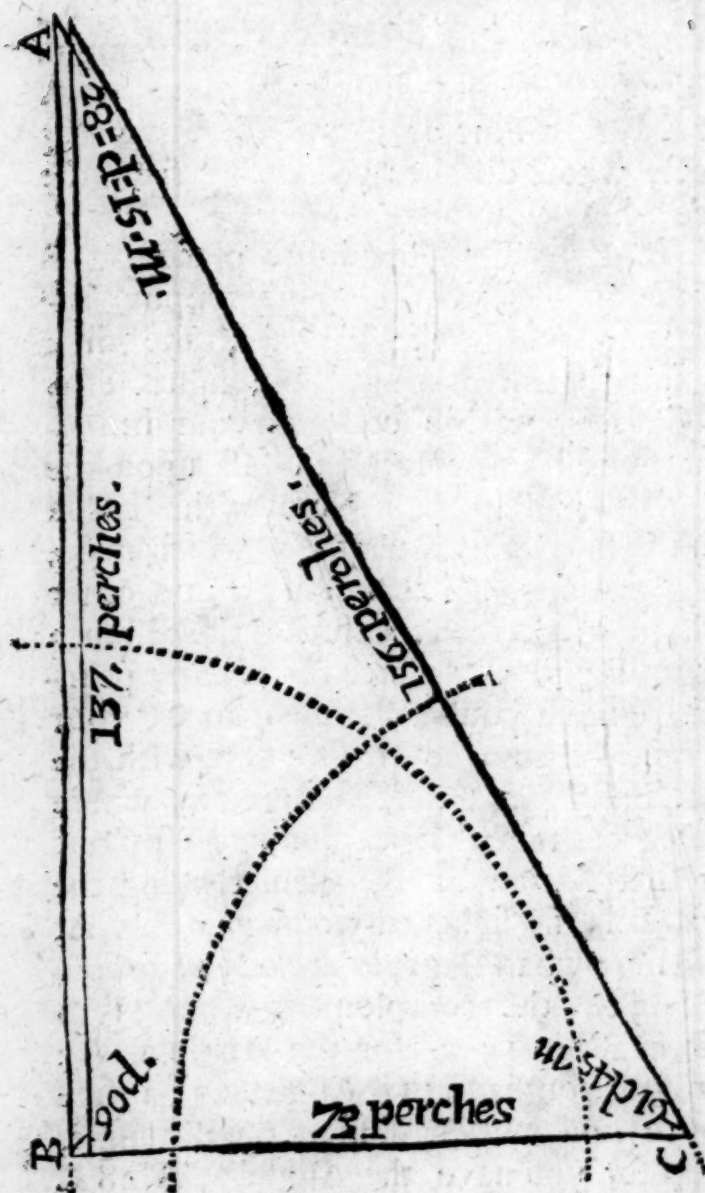
But to come to Practice, and by the Theodolite to find the Length or Distance of any place from you, how far soever it be.

This (as was formerly hinted) is no more but the resolving of a Triangle supposed to lye flat or Horizontally upon the Ground; as the taking of Heights is the
resol-

resolving of a Triangle, standing vertically or upright upon his Horizontal Base ; To shew you how to do it, take this Example.

Suppose you standing at B, and at A there were a Fort or Castle, or any other object, whose distance you desire from the place where you stand.

First, By your Theodolite placed at B lay the Index with the Sights upon the Diameter, where the Degrees take their beginning, and thorough the sights espy the Castle at A, the body of your Instrument so resting or screwed fast upon the staff that it stands on, move the Index with the sights to 90 degrees, which make a Right Angle. There espy some other mark, as at C, then measuring the distance betwixt B and C, (which suppose 73 perches) remove your Instrument to C, there with the Index and Sights upon the Diameter where the Degrees take their beginning, espy out your first station at B, then turning the Index with the sights till you again espy A, where note what Degrees are cut ; suppose 61 d. 45 m. the complement whereof to 90 d. is 28 d. 15 m. for the Angle at A : Therefore you have two Angles and a side given to find another side of the Triangle, ABC ; for you have the Angle at A 28 d. 15 m. and its opposite side BC 73 perches,



and again you have the Angle at C $61^{\circ} 45'$ *m.* which holds the same proportion to its opposite side BA, which is the distance required.

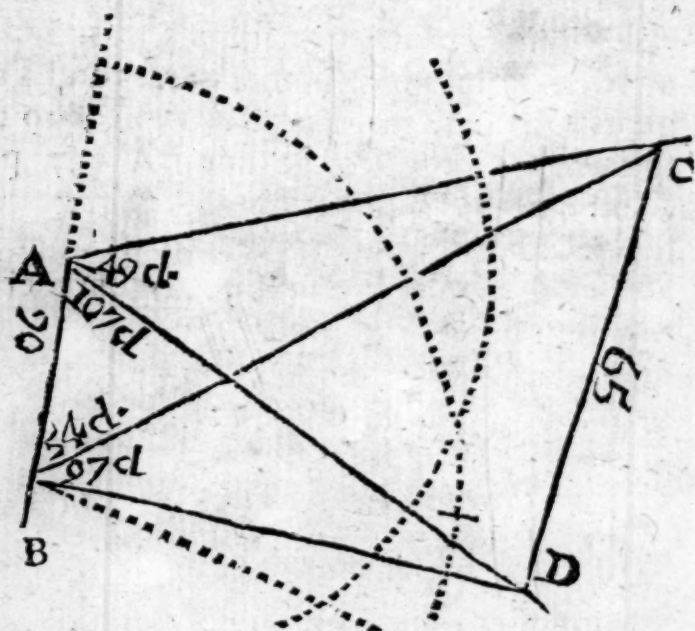
Which side or distance you may find by Resolving the Triangle either by Protraction, as in *pag.* 132, 133, or by Arithmetick working by Logarithmes, as *pag.* 31. or by the Lines of Numbers, Sines and Tangents, *pag.* 41. for as $28^{\circ} 15'$ *m.* is to 73, so is $61^{\circ} 45'$ *m.* to the side BA 137 perches: and (if the other side CA be required) so is Radius or 90 degrees to its opposite side by the general Rule in Trigonometry, *pag.* 141. being 156 perches.

S E C T. 2.

To find the distance betwixt any two Forts, and yet come near neither of them.

Suppose C one Fort, and D another, and you are to find the distance betwixt them, and come nearer neither of them than A and B. First place your Instrument at A, and laying your Index upon the Diameter where the Degrees take their beginning, through the sights espy the Fort at C, the Instrument so resting, move the Index till through the sights you espy D, which will be when the Index cuts upon the Limb 49

Degrees, which is the quantity of the Angle CAD ; Then moving your Index, till through the Sights you espy your second station at B, then you will find the Index to cut 107 Degrees, which is the quantity of the Angle CAB : Then measure the di-



stance betwixt A and B your first station and second, which suppose 30 ; then removing your Instrument to B, lay your Index with the sights upon the Diameter where the Degrees take their beginning, and move the Body of the Instrument upon the staff, till through the sights you espy the place of your first station A, there screw
your

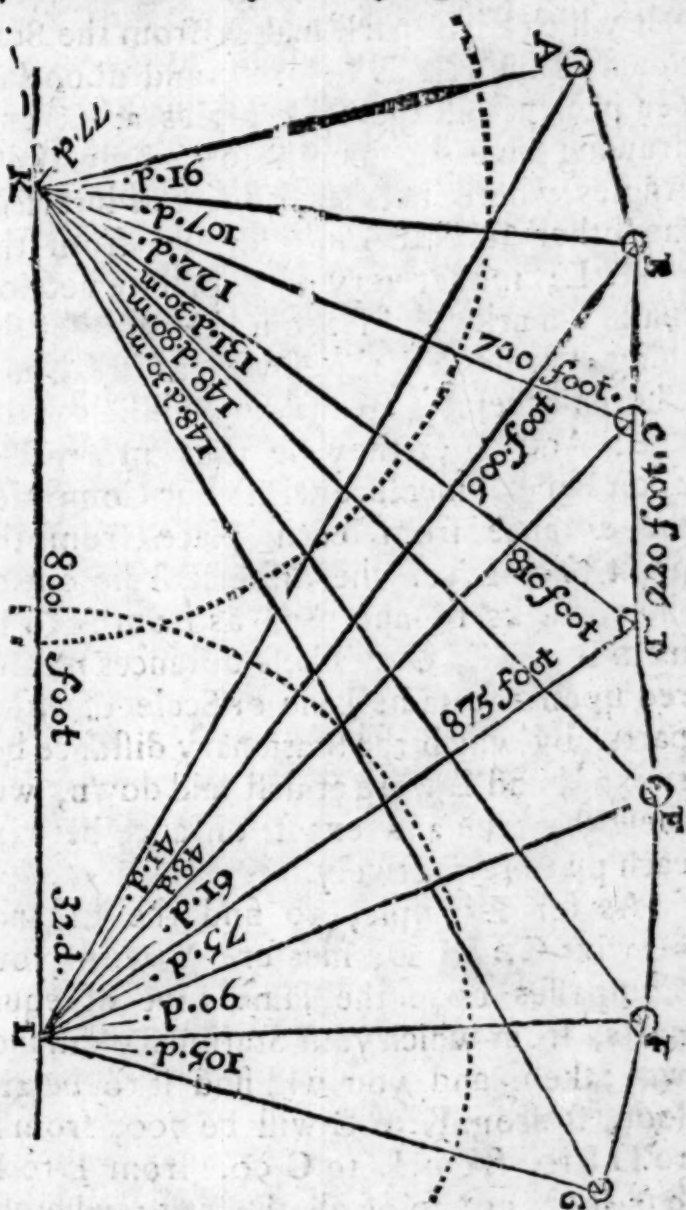
your Instrument fast, and move the Index till through the sights you espy the first Fort again at C, and observe how many Degrees are cut by the Index, which you will find 54 degrees, or the Angle ABC : Lastly, move your Index, till through the sights you espy the second Fort at D, and there observe what Degrees are cut by the Index, which you will find 97 degrees, or the Angle ABD. Note these several observations as you take them, and then upon Paper protract the same : As First, Draw your Stationary Line AB, and upon the point A protract the Angles 49 and 107 *d.* Likewise upon B protract the Angles 54 and 97 *d.* and lastly draw the line CD from the points of Intersection at C and D, and that measured upon the same Scale whence your Stationary distance was taken, will shew you the true distance betwixt the Forts C and D, which was the thing required.

S E C T. 3.

How to take the Distance of divers places one from another according to their true Scituation in Plano, standing at some convenient distance.

Make choice of two convenient Stations (taken as large as you can, whereby your

work will be the truer,) from which stations you may plainly discern all the marks or places whose Distances you would take : As suppose K and L to be your two Stations ; First then, place your Instrument at L, and turn the whole Instrument about, till through the sights on the Index lying upon the Diameter, (that is, where the Degrees take beginning) you espy your second station at K, which found, fasten your Instrument : Then moving your Index with the sights, till you espy the mark at A, noting how many degrees are cut by the Index to the end you may protract the same when you have done your observation, and after the same manner move the Index, till through the sights you espy all the rest of the marks one after another, as BCDEF, noting down the several Angles from the Stationary Line KL. Then removing your Instrument from L to K, measure the Stationary distance betwixt L and K, which suppose 800 foot, as in the Example, when you come to protract, prick out this distance from a Line of equal parts upon the line K L, and placing your Instrument at K, lay the Index upon the Diameter, and turn the whole Instrument about till through the back-sights you espy your first station L, and there screw it fast ; then moving the Index, direct your sight
so



so as to espy the several marks at ABCDE
FG, observing and noting the several An-
K 5 gle 3

gles which each mark makes from the Stationary Line KL as before ; and upon Paper protract all the said Angles at K, and drawing Lines at length to include those Angles, where these Lines meet or intersect the other several Lines drawn from the point L; in that very point of intersection make a mark, and set to it the name of the thing represented thereby ; as if A be a Church-Steeple, B a Wind-mill, or the like : And after they be thus protracted upon Paper, measure with your Compasses the distance from each place from the point K or L, or the distance from one of these marks to another, as from A to B, from B to C, &c. These distances measured upon the same Line or Scale of equal parts, by which the Stationary distance betwixt K and L were at first laid down, will shew the true and exact distance betwixt each place respectively.

As for Example, to find the distance betwixt C and D, measure it with your Compasses upon the same Line of equal parts, from which your Stationary distance was taken, and you will find it to be 220 foot, so from K to C will be 700, from K to D 810, from L to C 90, from L to D 875 foot, and so of all the rest ; whereby you may perceive this is the same with what is taught *pag.* 197 ; only that is taking the distance

distance betwixt 2 places only, and this betwixt many at the same time, and upon the same ground; and you may further discern, that these are no more but the resolving of Triangles by Protraction, wherein two Angles and a Side are given; the rest of the Sides and Angles (if need be) may be found as by protraction on this manner, so likewise by Arithmetical Operation also, which I leave on purpose to exercise the Readers Ingenuity, and pass on.

S E C T.

move in a perpendicular Line to CE, till you see the mark set up at B, and the point E in a Right Line, and set up another Staff at that place at D, getting the exact Distance thereof from C, which suppose 76; then subtract the measured Distance AB 66 from the measured Distance CD 76, and note the Remainder, which is 10. Then by the Line of Numbers, or by the Rule of Proportion.

As the Difference between AB and BC 10 is to the distance between A and C 50, so is the measured Distance CD 76 to the Distance between C and E 380. Or, so is the measured Distance AB 66 to the Distance AE 330.

S E C T. 5.

To take a Distance by a Carpenters, or Masons Square.

Take a Square with two Sights on one side, fasten it to the top of a Staff, which suppose 6 foot high, move the Square, so



that

that through the Sights you espy the thing which you desire to know how far it is distant from you : Then observe the distance betwixt the Staff (fastned to the other side of the Square where the sights are not) and the 6 foot staff which suppose 2 foot ; Then observe as often as that distance is contained in the length of the Staff, so many times is the length of the Staff contained in the Distance required. As how many times 2 in 6, that is 3 times. Then say, 3 times 6 the length of the Staff is 18, the distance required. As in the Example.

S E C T. 6.

To Measure an Inaccessible Distance, as the Breadth of a River with the help of ones Hat only.

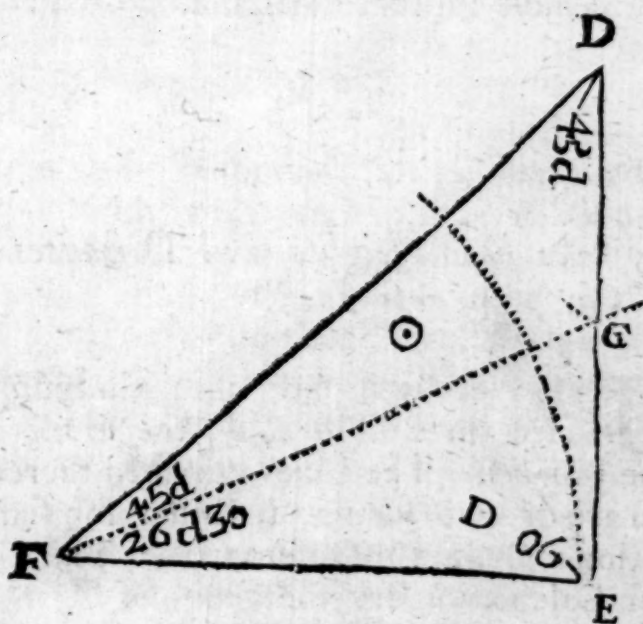
Having your Hat upon your Head, come near to the Bank of the River, and holding your head upright (which may be done by putting a small stick to some one of your buttons to prop up the Chin) pluck down the Brim or Edge of your Hat until you may but see the other side of the Water, then turn about the body in the same posture towards some plain, and mark where the sight by the brim of the Hat glanceth on the Ground : For the Distance from that place

place to your standing is the Breadth of the River required.

S E C T. 7.

How to take the Breadth of a River exactly.

Place your Theodolite, (or any other Instrument whereby you can take Angles ;) at E observe some mark on the other side the River, as at D, and observe an Angle



of 90 Degrees betwixt D and F, which may be either on the Right Hand, or on the Left, so far in the Line of 90 Degrees, till you

you espy the mark D doth justly make an Angle of 45 Degrees with the mark E, and this will be when you come to F; then measure carefully the Distance E F, and that shall be exactly equal to the Breadth of the River DE: An Angle of 26 *d.* 30 *m.* makes the Distance double to the Breadth, &c. and so by other Angles, but the best and surest is 45 *d.* if you can use no other: So likewise by the same Rule may be taken the Distance of any Cape or Island from you at Sea, or any other Accessible Height, or Inaccessible Distance at Land.

C H A P. IV.

To take both Heights and Distances by Gunters Cross-Staff.

THE necessary parts of this Instrument are these, The *Staff*, the *Cross*, and the 3 *Sights*. The Lines inscribed thereupon are of two sorts: 1. Either for Calculation: Or, 2. For Observation: The Lines for Calculation are the Lines of Artificial Numbers, Sines, and Tangents, whereof we have spoken largely. The Meridian Line for drawing of Sea-Charts; as also the Lines of Versed Sines, Chords, &c. which

which I shall say nothing of in this place. The second sort of Lines inscribed upon the Instrument are for Observation: First, A Line of Tangents for Observation of Angles. Secondly, A Line of equal parts not only for taking Angles, but also for taking perpendicular Heights and Distances. The Tangent Line on the Staff may be known by the double Numbers set on both sides of the Line, beginning at one side at 20, and ending at 90, and on the other side at 40, and ending at 180. On the Cross there are two Tangent Lines.

1. A Tangent Line of 36 *d.* 3 *m.* numbered by 5, 10, 15, to 35, the midst whereof is at 20 *d.* and therefore I call it the Tangent of 20, and this hath respect unto 20 *d.* in the Tangent on the Staff.

2. A Tangent Line of 49 *d.* 6 *m.* numbered by 5, 10, 15, to 45, the midst whereof is at 30 *d.* and hath Respect to 30 *d.* in the Tangent on the Staff, and therefore called the Tangent of 30. The Use of these follows.

S E C T. I.

To find an Angle by the Tangent on the Staff.

Let the middle Sight be always set to the

the middle of the Cross, noted with 20 and 30, and then the Cross drawn nearer the Eye, until the marks may be seen close within the sights: For if the Eye at A (the end of the Staff) which is noted with 90 and 180, beholding the marks K and N between the two first sights C and B, or the marks K and P between the two outward sights, the Cross being drawn down to H, shall stand at 30 and 60 in the Tangent of the Staff: it sheweth the Angle KAN is 30 d. the Angle KAP 60 d. the one double to the other, which is the reason of the double Numbers on this Line on the Staff; and this way will serve for any Angle from 20 towards 90 d. or from 40 to 180 d. but if the Angle be less than 20 d. we must then use the Tangents on the Cross.

S E C T. 2.

To find an Angle by the Tangent of 20 on the Cross.

Set 20 unto 20, that is, the middle sight to the midit of the Cross at the end of the Staff noted with 20, so the Eye at A beholding the marks L and N close between the two first sights C and B, shall see them in an Angle of 20 degrees: If the marks shall be nearer together as are M and N, then

then draw the Cross in from C to E: If they be further asunder, as are K and N, then draw out the Cross from C to F, so the Quantity of the Angle shall be still found in the Cross in the Tangent of 20 *d.* at the end of the Staff; and this will serve for any Angle under 20, and so to 35 Degrees.

S E C T. 3.

To find an Angle by the Tangent of 30 upon the Cross.

This Tangent of 30 is here put the rather, that the end of the Staff resting at the Eye, the Hand may more easily remove the Cross: For it supposeth the Radius to be no longer than AH, which is from the Eye at the end of the Staff to 30 *d.* about 22 Inches, and 7 Parts, wherefore here set the middle sight to 30 *d.* on the Staff, and then either draw the Cross in or out until the marks be seen between the two first sights: So the quantity of the Angle will be found in the Tangent of 30, which is here represented by the Line GH, and this will serve for any Angle from 0 *d.* towards 48 degrees.

S E C T.

S E C T. 4.

To take the Altitude of the Sun by Thread and Plummet.

Let the middle sight be set to the midst of the Cross, and to that end of the Staff which is noted with 90 and 180; then having a Thread and Plummet at the beginning of the Cross, and turning the Cross upwards, and the Staff towards the Sun, the Thread will fall on the complement of the Altitude above the Horizon, and this may be applyed to other uses also. Thus much for the Line of Tangents.

The other Line for Observation, whereby you may find both Angles and Sines is the Line of equal parts, which in *Gunters* upon the Staff is 35 Inches, each Inch divided into Tens, and each Ten into Halfs; and to answer it upon the Cross is $26\frac{1}{2}$ Inches between the two outward sights.

S E C T. 5.

To take Angles by these Lines of equal parts.

If the Angles be observed between the two first Sight, there will be such proportion between the parts of the Staff, and the

the parts of the Cross, as between Radius, and the Tangent of the Angle ; as if the parts intercepted on the Staff be 20, the parts of the Cross 9 : Then by Proportion, as 20 is to 9 upon the Line of Numbers, so is the Radius to the Tangent of 24 d. 14 m. upon the Line of Tangents : But if the Angle be observed between the two outward Sight, the parts being 20 and 9, as before, the Angle will be double to the former, viz. 48 d. 28 m.

S E C T. 6.

To take not only Angles, but Sides also by this Line of Equal Parts, which is, to take Heights and Distances.

And First, For Heights.

In taking of Heights you are to hold the Cross upwards, and for Distances, to hold the Cross sideways, and the Eye placed at the beginning of the Staff, espying the marks by the inward sides of the Sight, there will be such proportion betwixt the Distance and the Height as is between the parts intercepted on the Staff and Cross : The Ground whereof you will find in pag. 189.

P R O B L.

P R O B L. I.

Having the Distance from your standing to the foot of the Altitude given, to find the Height.

As the Segment of the Staff is to the Segment on the Cross, so is the Distance to the Height. As if the Distance AB being measured, or known to be 256 feet, it were required to find the Height BC. First, I place the middle sight at 24 Inches (or Parts,) then holding the Staff level with the Distance, I raise the Cross parallel to the Height or Top of the Altitude; so as my Eye may see from A, (the beginning of Inches, or Parts upon the Staff) by the sight E unto the mark C; which being done, I find 18 Inches intercepted on the Cross between the sights at E and D: Therefore I say, the Height BC is 192 feet; for as 24 is to 18, so is 256 to 192: Or if the observation were to be made before the Distance were measured, I would set the middle sight to 12, 16, 20, 24, or some such Number as might best be divided into several parts, and then work by Proportion: As in the former Example, Suppose I set the Staff to 24, and the Cross to 18, it shews the Height is $\frac{3}{4}$ of the Distance, therefore the Distance being 256, the Height must be 192 as before.

PROBL.

P R O B L. 2.

Having part of the Height given, to find the whole Height.

As if the Height from G to C were known to be 48, and it were required to find the whole height BC. First, put the third sight, or some other sight upon the Cross between the Eye and the mark G. Then, as the Difference between the sights E and F 45, is to the whole Segment on the Cross ED 180, so is the Part of the Height given GC 48 to the whole Height BC 192.

P R O B L. 3.

To find an inaccessible Height at 2 Stations, by knowing only the Stationary Distance.

As the Difference of Segments on the Staff is to the Stationary Distance, so is the Distance on the Cross to the Height required: As suppose the first Station at H, the Segment on the Cross ED were 180, and the Segment of the Staff HD 300. Then coming 64 Feet nearer to B in a direct Line towards the Height unto a second station at A, and making another observation: Suppose the Segment of the Cross

Cross ED were still 180 as before, and the Segment of the Staff AD 240. Take 240 out of 300, the Difference of Segments will be 60: Then say,

As 60 the Difference of Segments to 64 the Stationary Distance (or Difference of Stations) so is DE 180 the Segment of the Cross to the Height required, viz. BG 192.

P R O B L. 4.

Having the Height of any thing given, to find the Distance.

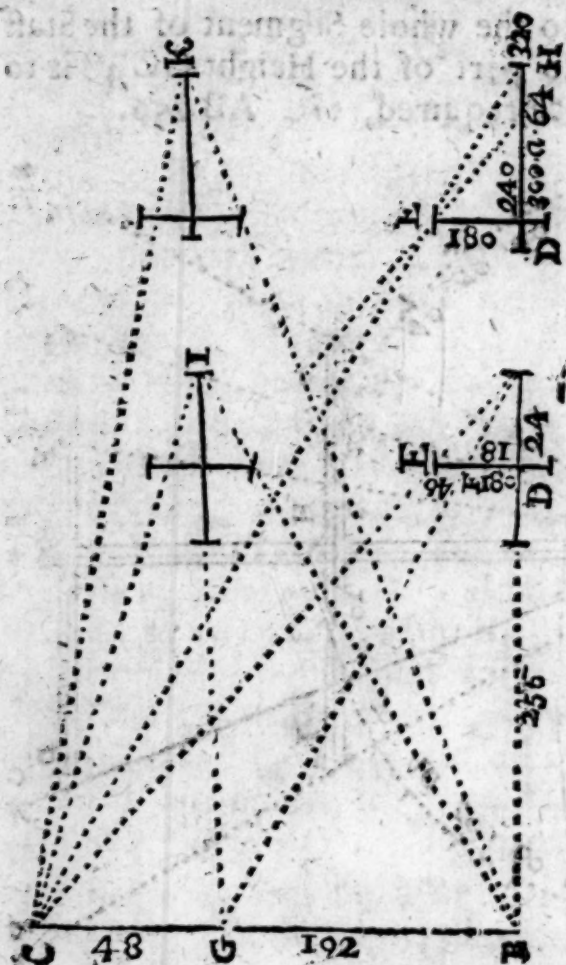
Secondly, For finding Distances.

As the Segment of the Cross (or the parts upon the Cross which is all one) is to the Segment of the Staff, (or the parts upon the Staff,) so is the Height given to the Distance required. For Example, as the Segment ED 18 on the Cross is to DA 24 on the Staff, so is CB 192 the Height to 256 the Distance.

P R O B L. 5.

Having Part of the Height given, to find the Distance.

As the Difference between the sights
EF 45



the whole Segment of the Staff to the distance required. The Segment of the Cross being as before 180. The Segment of the Staff at the first Station 240, at the second 300. The difference between these 60. The distance between the two Stations 64. Then say,

As

As 60 the difference of Segments is to 64 the Stationary distance, so is the Segment at the first station 240 to the distance AB the first station 256. And so is 300 the Segment at the second Station, to 320, the distance HB the second Station.

In taking a Breadth it is not material whether the two Stations be chosen at one end of the Breadth proposed, or without it, or within it; if the Lines between the Stations be perpendicular unto the Breadth; as may appear if instead of the Stations at A and H, we make choice of the like Stations at I and K. To conclude, in all these there is regard to be had to the parallax of the Eye, and his Height above the Horizon; to the Semidiameter of the Sun, his Parallax and Refraction: As also the Parallax and Refraction of the Moon and Stars, if you will be so very exact; which you may see in this following Table.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

The Refraction of the Sun, Moon, and Stars causes them to appear higher above the Horizon than they really are, therefore the Refraction from the Altitude observed, that the true Altitude may be had.

The

The Table of these Parallaxes according to the Observations of *Ticho Brahe*.

The Table of Refractions of the Sun, Moon, and Stars.

Altitudes.	Sun min.	Moon min.	Stars min.	Altitudes.	Sun min.	Moon min.
0	34	33	30	18	6	6
1	26	25	21	19	5	6
2	20	20	15	20	4	5
3	17	17	12	21	4	5
4	15	15	11	22	3	4
5	14	14	10	23	3	4
6	13	13	09	24	3	4
7	12	13	8	25	2	3
8	11	12	7	26	2	3
9	10	11	6	27	2	3
10	10	11	5	28	2	2
11	9	10	5	29	2	2
12	9	09	4	30	1	2
13	8	09	4	31	1	2
14	8	8	3	32	1	1
15	7	8	3	33	1	1
16	7	7	2	34	1	1
17	6	7	2	35	1	1

The Refraction of the Sun, Moon, and Stars causeth them to appear higher above the Horizon than they really are, therefore Subtract the Refraction from the Altitude observed, that the true Altitude may be had.

The

The cause of these Refractions or Parallaxes is the Atmosphere, or the gross Vapours and Thickness of the Air near the Horizon, or the Face of the Earth; which to demonstrate by an easie experiment, set down an empty Bason on a Stool, laying a shilling in the bottom of it; and go so much backwards, till you bring the edge of the Bason and the Shilling in a right Line with your Eye, and then let another fill the Bason with fair Water, and you may go a pretty way further backwards, and still see the Shilling and the Edge of the Bason in a right Line, the Refraction of the Water being the Cause thereof; and from this Reason we conclude that the Sun seems to appear above the Horizon when he is really set; and that other times he seems higher than he really is, as also the Moon and Stars.

C H A P. V.

HAVING shewed you several ways to take Heights and Distances in general, I come now to speak of *Surveying*, as it is taken in a strict and particular sense for the Measuring of Land according to the Diversity of Inclosures: Now this Measuring of

Land is no more but finding the Area or Content of a Superficies, and consists in these two Particulars. First, Plotting of Ground, or taking the Figure of it: Secondly, the Measuring of that Plot or Figure so taken, or taking the Superficial Content thereof.

1. The former of these, *viz.* The plotting of ground is excellently performed: First, By the Plain Table. Secondly, By the Theodolite: Or, Thirdly, By the Chain only, and Scale and Compass.

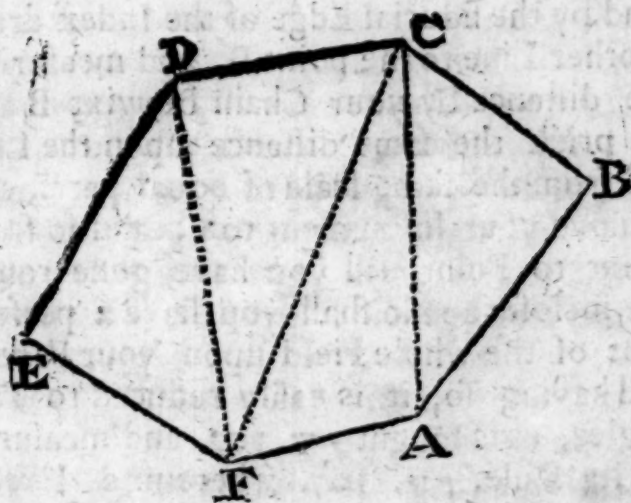
2. The latter, *viz.* The Measuring of a Plot is no more but the reducing of it to one or more of the Three Regular Geometrical Figures before-mentioned, *viz.* either to a *Circle*, a *Square*, or a *Triangle*, (especially the last of these) and measuring it by their Rules, seeing no Plot or Figure can be so Irregular, but may be reduced to Triangles, (if not to either of the former) and so may be measured by *Trigonometry*.

S E C T. II.

First then, To take a Plot by the Plain Table.

A Plain Table is no more but a Board, or rather three pieces put together, making a long Square or Parallelogram, sufficient to hold a sheet of Paper, with Ledges
to

to keep the Table together, and the Paper fast, with a loose Index with Sights to lye upon the Table: The use is as follows. First, Placing your Instrument upon the Staff, fasten it by the screw in the socket, which it hath common with a Theodolite, and standing at any Corner of the Inclosure, whose Plot you would take as at A, lay the Index with the sights upon the Paper, and thorough the sights espy such a mark upon the Hedge or Side of a Field or Inclosure as far as the Hedge goes in a straight Line as at B; and by the Fiducial



edge of the Index, draw a Line what length you please from A towards B: Then removing your Instrument measure with your Chain the distance betwixt A and B, and

from a Scale of equal parts prick down in Perches the said Distance from A to B upon the Line drawn; then placing your Instrument at B, lay the Index upon the Line AB, and turn the whole Instrument till through the sights you espy A your first station; there screw the Instrument fast upon the Staff, observing that A lay even with the Line drawn, then moving the Index to the Point B, keep one end thereof to that point; and move the other till through the sights you espy a third mark at C, (which again is as far as the Hedge or Side of the inclosure goes in a right Line :) And by the fiducial Edge of the Index draw another Line to the point B, and measuring the distance by your Chain betwixt B and C, prick the same distance upon the Line BC from the same scale of equal parts, and remove your Instrument to C, and so from Point to Point, till you have gone round the inclosure: So shall you have a perfect Plot of the whole Field upon your Paper; and having so, it is easily reduced to Triangles, as is taught *pag.* 102. and measured by its Rule, *pag.* 107. whereunto I refer the Reader for brevity.

S E C T. 2.

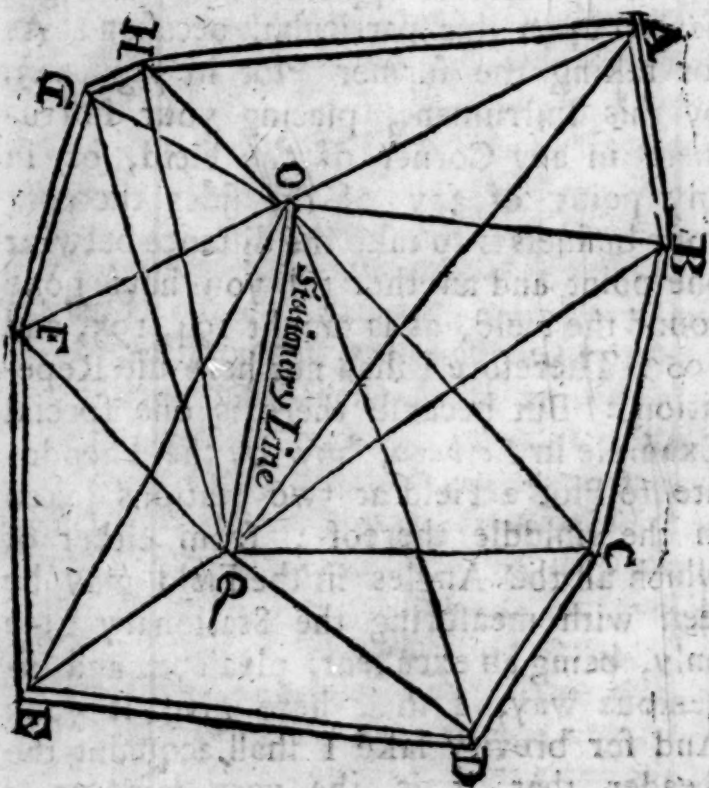
Secondly, To take a Plot by the Theodolite.

The Description of a Theodolite you have pag. 183, and 194. The use of it in taking distances in general, you have also pag. 194, 195, 196, 197, 198; and as for measuring of Land thereby, it is no more but the particular application of those generals upon this particular occasion: As for taking the former Plot in pag. 223. by this Instrument, placing your Instrument in any Corner of the Field, or in any point of any of the sides thereof; your business is to take the distance betwixt one point and another till you have gone round the Field, as is taught 194, 195, and 196: Therefore I shall not here use Repetition: But because there is one special Example in *Leyburn*, how by the Theodolite to Plot a Field at two Stations taken in the middle thereof; from either of which all the Angles in the Field may be seen with measuring the Stationary Line only, being an excellent, pleasant, and ingenious way, I shall here give it you: And for brevity sake I shall acquaint the Reader that it is the very same as is shewed pag. 200, and 201, KL being

L 5

supposed

Supposed the Stationary Line in the middle, and the whole Figure in *pag.* 120, being supposed to be only one half of the Plot, and Lines drawn from one point to another round the whole, as is there done; by which means you will have the Periphery of the whole Plot; but to give you an entire Example or Figure more, referring you to the former place for Direction, observe this following.



S E C T.

S E C T. 3.

Thirdly, To take the Plot of a Field without any other Instrument but the Chain only, and by your Scale to Protract the same upon Paper.

Let ABCDEF be a Field to be plotted, First, Measure every side thereof with your Chain beginning at A, so shall you find, The side AB contains 2 Chains 5 Links

BC	1	48
CD	2	6
DE	4	75
EF	2	15
FA	2	52

Note that these are measured by Mr. Gunter's Chain (which I do most commend, and which contains 4 Poles, each Pole 25 Links, or 100 Links in the whole Chain, each Link contains 7 $\frac{1}{2}$ Inches, in all 792 Inches, 66 Foot, or 22 Yards.)

Now after that you have taken or measured these several sides by this Chain, to Protract the same upon Paper, observe, That the Field contains 6 sides, and therefore may be reduced into two Trapezia's, or into 4 Triangles. A Trapezia is any Figure of 4 sides, whose sides and Angles are all unequal, so that this field may be reduced

ced into the Trapezium ABCF, and FCED by a Line drawn from the Angle F, to the Angle C, which being measured by your Chain, you will find the Length of it to be 2 Chains 89 Links. Then again, if you measure with your Chain from C to A, which will contain 2 Chains, 70 Links, and from C to E, which contains 3 Chains, 60 Links; you will find the former two Trapezia's divided each into two Triangles, *viz.* the Trapezium ABCF into the Triangles ABC and ACF: And the other Trapezium FCED into the Triangles FEC and CDE; in every which Triangle you have all the sides given, by means whereof you may draw upon Paper the exact Figure of your Field by what Scale you please, (according to the Rule given, *pag.* 85.) and as it here follows.

First, Taking into your consideration the Scituation of your Field, draw upon your Paper the Line AC, containing 2 Chains, 70 Links of any scale: Then because the side AB contains 2 Chains, 5 Links, open your Compasses to 2, 5, and placing one foot in A, with the other describe the pricked Arch *a b*. Likewise take in your Compasses 1, 48 the length of BC and placing one Foot of your Compasses in C, with the other describe the pricked Arch *c d*, crossing the former Arch *a b* in the

the point B, then drawing 2 Lines from C and A to the point B, you have protracted the Triangle ABC: After the same manner must you protract the Triangles AFC, FEC, and CED, so is your work finished, and the Figure of the Field protracted upon Paper.

C H A P. VI.

Now for to know how to Measure this Plot thus protracted, or any of the 2 former, pag. 223, 224, and by consequence any other. Take these Rules following.

S E C T. I.

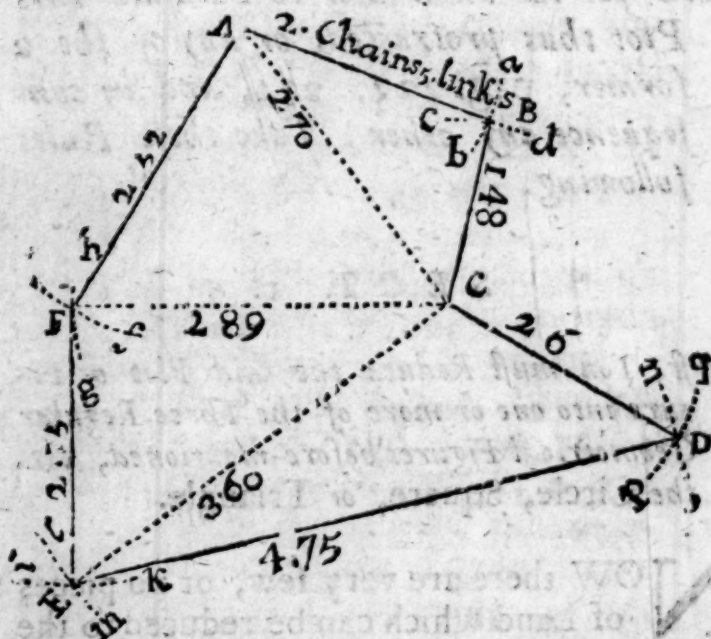
First, You must Reduce the said Plot or Figure into one or more of the Three Regular Geometrical Figures before-mentioned, viz. the Circle, Square, or Triangle.

NOW there are very few, or no pieces of Land which can be reduced to the first, viz. the Circle, but what may be done as to that you have pag. 74: But to the second, viz. the Square, many sorts of

of Figures may be reduced thereunto, as you may see *pag.* 89, 90. But in the third and last place, any irregular Plot whatsoever may be reduced to Triangles, as you may see *pag.* 104 and 105, and as you see this Plot is done.

S E C T. 2.

Now this being done, there is no more to be done, but to measure these several Triangles by letting fall perpendiculars, as



is taught *pag.* 55, and multiplying half the Base, and the whole Perpendicular, on the whole

whole Base and half the Perpendicular together, which will give the superficial Content of each Triangle, and the Contents of the several Triangles added together, will give the Content of the whole Plot in Acres, Roods, and Perches.

R U L E 1.

And for finding the Superficial Content of a Triangle, having all the Three sides thereof given, it is taught *pag.* 113. which may be done without letting fall any perpendicular: For the doing whereof I refer the Reader to the place before-mentioned, *pag.* 113.

R U L E 2.

The Second Rule is, To find how many Acres, Roods and Perches are contained in any Triangle, or any other piece of ground of any other shape. And for this purpose note, that 3 Barley Corns is an Inch, 12 Inches a Foot, 3 Foot a Yard, &c. And that by the Statute $5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ Foot make a Perch or Pole, 40 square Perches make a Rood, and 4 Rood an Acre, so that 160 square Perches are an Acre: Suppose a piece of Ground should contain 917 square Perches, Divide 917 by 40, the Quotient will

will be $22 \frac{1}{40}$, divide this again by 4, the Quotient is 5, 2, 37, which is 5 Acres, 2 Roods, 37 Perches; or divide 917 by 160, the Quotient will be 5 Acres, and 117 Perches, which is 2 Roods, 37 Perches, as before.

R U L E 3.

Note again to bring Chains and Links into Acres, Roods, and Perches: That 100 Links (of Mr. Gunter's) make a Chain, 10 Square Chains an Acre. Now suppose a Piece of Land to contain 9 Chains, 50 Links in length, and 6 Chains 25 Links in breadth, lying in a long Square: Multiply the one by the other, or 950 Links by 625, the Product is 593750, from which Product in Square Links cut always off the five last Figures, so is the rest to the left hand compleat Acres, which is here 5. Those which are cut off being 93750 multiplied by 4, (the number of Roods in one Acre) produceth 375000, from which Product cutting off again the 5 last Figures towards the Right Hand, the Remainder towards the Left Hand shews the compleat Roods, which are here 3: The 5 Figures cut off being 75000, multiplied by 40 the Number of Perches in a Rood, produceth 3000000; so cutting off the 5 last Cyphers towards the

the Right hand, leaves 30 towards the Left, which are 30 Perches : If any odd Numbers had remained in those which were cut off, they had been so many Hundred Thousand parts of a Perch ; but no Fraction remaining, the Contents of this Parallelogram or Long Square, is 5 Acres, 3 Roods, and 30 Perches : the like is to be done in any other.

S E C T. 3.

But to help you herein, observe the Table following as to the former Example 593750. Cutting off the 5 last Figures, there remains 5 Acres : Then look for 90000 in the Table under the Title Links, and against it under the Title Roods and Perches, you find 3 Roods, 24 Perches : Then look for 3750, and against it you find 6 Perches, all which added together gives you the Area or Contents of the whole, as before ; 5 Acres, 3 Roods, and 30 Perches. The like is to be observed in any other Number of Square Links, to reduce them to Acres, Roods, and Perches.

5. A.	0. R.	0 P.
_____	3 _____	24
_____	:: _____	6
_____	_____	_____
Somma 5 _____	3 _____	30

Square

Square Links.	R.	P.	Square L.	R.	P.
100000	4	00	5625	—	9
90000	3	24	5000	—	8
80000	3	08	4375	—	7
70000	2	32	3750	—	6
60000	2	16	3125	—	5
50000	2	00	2500	—	4
40000	1	24	1875	—	3
30000	1	8	1250	—	2
20000	0	32	625	—	1
10000	0	16			
9375	—	15	Note that 220 yds. or 11 Score in Length, and 22 yds. in Brea. makes an Acre.		
8750	—	14			
8125	—	13			
7500	—	12			
6875	—	11			
6250	—	10			

S E C T. 4.

Having the Length of any piece of Ground in Perches, to find how many Perches in Breadth will make an Acre.

As the Length in Perches (which suppose 50) is to 160 upon the Line of Numbers, the same extent the same way will reach from 1 to the Breadth in Perches in this Example, 3 $\frac{20}{100}$, or 3 $\frac{1}{5}$ part of a Perch. Di-
vide

vide 160 by 50, Quotient $3\frac{1}{5}$, which is the same. But in Chains and Links : As the length in Chains is to 10, so is one Acre to the Breadth in Chains and Links : As the Extent from 12 Chains 50 Links to 10, will reach from 160 Links, or 1 Acre to 80 Links the Breadth required. *Leyburn pag. 276.*

S E C T. 5.

To know whether you have taken the Angles of a Field truly.

Add all the Angles together, and if the Sum of them all do agree with the Sum of 180 degrees multiplied by a Number less by two than the Number of Angles in the Field, your work is true, otherwise not : This is where all the Angles are inward, but if any of the Angles be outward, then take the Complements of such Angles to 180 Degrees, which is to be added to the rest of the Angles instead of the Angles themselves, and these outward Angles (if never so many) must not be accounted as Angles in the Multiplication, but wholly rejected.

To reduce Chains and Links (of Gunter's Chain) into Feet and Yards.

Set down the Number of Chains and Links as one whole Number, (as suppose 5 Chains 32 Links, set them down 5.32) multiply this 532 by 66 the Product is 35112, from whence cut the two last Figures towards the Right hand, so shall the Figures to the Left hand remaining be Feet, and the Figures cut off 100 parts of Feet, viz. 351 $\frac{12}{100}$ Feet, these divided by 3 brings them into yards, viz. 117 yards. Or multiply 532 by 22 is 11704, and cut off the two last Figures, brings it into yards at once, viz. 117 as before, and $\frac{4}{100}$ parts of a yard the Fraction. But let the Number of Chains be what they will, if the number of Links be less than 10, you must place a Cypher before the number of Links, as suppose 9 Chains 5 Links, place them thus, 905, then multiply them by 66, as in the other Example, will produce 59730, from which cut off the two last Figures, and there will appear to be 597 Feet, $\frac{30}{100}$ parts, and so of any other : As for Example.

9.05

66

5430

5430

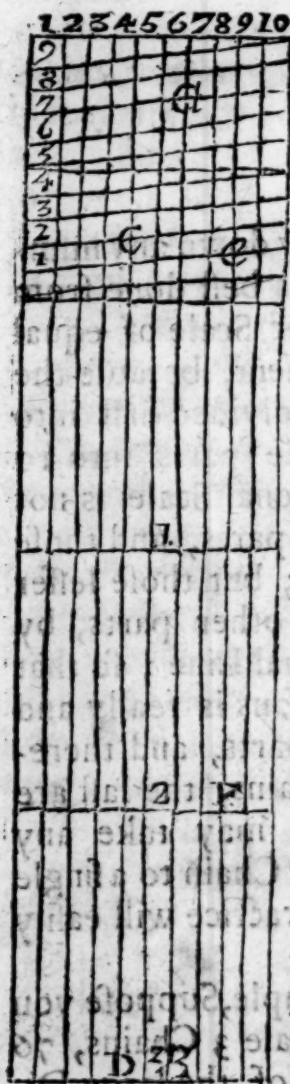
597.30



Note further, that to lay down any number of Chains and Links, is best done from a Diagonal Scale, any other Scale of equal parts not being so convenient, because the ordinary Scales are only divided first into 10 parts, and one of those parts into 10 lesser parts; but the Diagonal Scale is not only first divided into 10 parts, and those again into 10 lesser parts; but those lesser parts also divided into 10 other parts, by means of the cross Diagonal Line; so that each of the largest Divisions is really and visibly divided into 100 parts, and therefore taking the first for Chains, the last are Links, and out of it you may take any Number measured by your Chain to a single Link, to which a little practice will easily bring you.

As to give you one Example, Suppose you were to take from your Scale 3 Chains, 76 Links. First, Take three of the large Divisions in your Compasses for the three Chains;

Chains: Then for the Links observe, that the small Diagonal Divisions are numbred from 1 to 10 upon the Top, and likewise from 1 to 10 upon the



side, now reckon upon the side upwards from 1 to 7, and those signifie Tens, and run along that Line till you come just under 6 on the Top, and those signifie Units, and where those two Lines meet, there is the place signifying 76. Therefore the Extent of your Compasses from *a* to *B* is exactly 3 Chains, 76 Links, and from *D* to *C* is 3 Chains, 24 Links, and from *e* to *F* is 2 Chains 18 Links; and so of any other number that you have occasion to protract, be the same more or less. The Rule is the same always, counting upon the side upwards for Tens, & from the left side towards the right you must reckon Unites, and observing to

measure Chains and Links always upon the same

same parallel Line, which is drawn from one end of the Scale to the other.

S E C T. 6.

To take the Plot of any Champian Field of 2000, or 3000 Acres by the Plain Table, and never change Paper.

Place your Instrument at every Angle, and so get every Angle and his sides, not regarding the length of the containing sides as you be wont. Then Measure every Hedge, and as you use to lay the same down by your Scale and Compass, here you shall but write the length of every Hedge upon the Lines drawn upon your Paper, so they need not run off the same, because you may draw them as long and as short as you please, but when you come at home you must protract them according to due proportion.

S E C T. 7.

Angles taken by the Intersection of Lines often fall out so acute, that it is hard to find the true point of Intersection.

Therefore when you have taken a Field at two Stations by intersection of Lines (as is taught, pag. 223.) Then from the first Sta-

Station beyond the second, or from the second Station beyond the first, observe some Mark or Tree inclining towards the proposed Field (which let make rather a Right Angle than obtuse Angle with your Stationary Line) and so get the Angles it makes with both your Stations, making your Stationary Line one side of the said Angles : Then go to the mark espied for a third Station, and there observe such Corners that you thought would fall out Acute by Sections of Lines issuing from the first Stations, and thereby get the Angles they make with your first and second Stations, for by help thereof you shall correct the acute Section of the former Lines.

And note, that where a Field lies so that you cannot from 2 Stations see all the Corners thereof, what you cannot do at the first and second station, you may do with as many more Stations as need requires, as a third, fourth, or fifth Station, or more, as you see cause.

S E C T. 8.

To take the Plot of a Wood, or Waterish Ground, when by Reason of some obstruction or other, you cannot come within it.

Your way is to go round about the same,
and

and make your observations at every Angle, the Distance between Angle and Angle measured by your Chain will be the sides of the Plot, and protracting both the Angles and Sides upon Paper, will give you the true Figure of the whole, which you may measure by the Rules aforegoing.

S E C T. 9.

Take notice, that the most, or however, very many small inclosures are Trapezia, that is, having 4 unequal Sides, and 4 unequal Angles: Now any of these may be easily reduced into two Triangles, and so measured as I have formerly shewed you: But for the more ready casting up the Contents of these, note, that the Diagonal Line drawn from Corner to Corner, whereby the Trapezia is divided into 2 Triangles, is by that means the common Base to them both. Therefore add the two perpendiculars together, the half of which multiplied by the whole Base, the Product will shew the Contents of the whole Trapezia; or if you add both the perpendiculars together, and by that multiply the whole Base; half of the Product will be the Content of the Trapezia.

S E C T. 10.

Another way to measure a Field of 4
M unequal

unequal sides is this: First, Measure the top and the bottom, and add those two together, and take the half, then measure the two opposite sides, adding them also together, and take the half, these two halves multiplied the one by the other, the Product will give you the true Contents: As suppose one side be 16 Perches, its opposite side 6, these added, makes 22, the half whereof is 11: Then suppose one of the other sides 28, its opposite side 18, these added is 46, the half whereof is 23, which multiplied by 11 shews 253 to be the whole Content. Note, that to protract a Trapezia, you must by the Theodolite, or some such like Instrument observe one of the Angles, which being known and protracted, prick off the 2 containing sides, and from the end, or the Extremities of those 2 sides, you may draw the other two sides by 2 Arches, crossing one another at their Respective Distances, as in a Square or Triangle, as is elsewhere taught.

A TABLE

A TABLE of FRACTIONS.

O R,

Parts of Integer Pearches, that is, in any Number of Pearches, so far as the Table Extends. What a Quarter of a Pearch awanting, or over, will amount unto in Day-works, Pearches, Yards, Foot, and Inches Square.

A Quarter of a Pearch in Pearches.	P.	DP.	Y.	F.	I.	A Quarter of a Pearch in Pearches.	P.	D.	P.	Y.	F.	I.
	1	0	0	1	1 $\frac{1}{4}$		21	1	1	1	1	1 $\frac{1}{2}$
	2	0	0	2	2 $\frac{3}{4}$		22	1	1	2	2	3
	3	0	0	4	4 $\frac{1}{2}$		23	1	1	4	0	4 $\frac{1}{2}$
	4	0	1	0	0		24	1	2	0	0	0
	5	0	1	1	1 $\frac{1}{4}$		25	1	2	1	1	1
	6	0	1	2	2 $\frac{3}{4}$		26	1	2	2	2	3
	7	0	1	4	4 $\frac{1}{2}$		27	1	2	4	0	4 $\frac{1}{2}$
	8	0	2	0	0		28	1	3	0	0	0
	9	0	2	1	1 $\frac{1}{4}$		29	1	3	1	1	1 $\frac{1}{2}$
	10 is	0	2	2	2 $\frac{3}{4}$		30	1	3	2	2	3
	11	0	2	4	4 $\frac{1}{2}$		31	1	3	4	0	4 $\frac{1}{2}$
	12	0	3	0	0		32	2	0	0	0	0
	13	0	3	1	1 $\frac{1}{4}$		33	2	0	1	1	1 $\frac{1}{2}$
	14	0	3	2	2 $\frac{3}{4}$		34	2	0	2	2	3
	15	0	3	4	4 $\frac{1}{2}$		35	2	0	4	0	4 $\frac{1}{2}$
	16	1	0	0	0		36	2	1	0	0	0
	17	1	0	1	1 $\frac{1}{4}$		37	2	1	1	1	1 $\frac{1}{2}$
	18	1	0	2	2 $\frac{3}{4}$		38	2	1	2	2	3
	19	1	0	4	4 $\frac{1}{2}$		39	2	1	4	0	4 $\frac{1}{2}$
	20	1	1	0	0		40	2	2	0	0	0

P.	D.	P.	Y.	F.	I.	P.	D.	P.	Y.	F.	I.
41	2	2	1	1	$1\frac{1}{3}$	61	3	3	1	1	$1\frac{1}{2}$
42	2	2	2	2	3	62	3	3	2	2	3
43	2	2	4	0	$4\frac{1}{2}$	63	3	3	4	0	$4\frac{1}{2}$
44	2	3	0	0	0	64	4	0	0	0	0
45	2	3	1	1	$1\frac{1}{2}$	65	4	0	1	1	$1\frac{1}{2}$
46	2	3	2	2	3	66	4	0	2	2	3
47	2	3	4	0	$4\frac{1}{2}$	67	4	0	4	0	$4\frac{1}{2}$
48	3	0	0	0	0	68	4	1	0	0	0
49	3	0	1	1	$1\frac{1}{2}$	69	4	1	1	1	$1\frac{1}{2}$
50 is	3	0	2	2	3	70	4	1	2	2	3
51	3	0	4	0	$4\frac{1}{2}$	71	4	1	4	0	$4\frac{1}{2}$
52	3	1	0	0	0	72	4	2	0	0	0
53	3	1	1	1	$1\frac{1}{2}$	73	4	2	1	1	$1\frac{1}{2}$
54	3	1	2	2	3	74	4	2	2	2	3
55	3	1	4	0	$4\frac{1}{2}$	75	4	2	4	0	$4\frac{1}{2}$
56	3	2	0	0	0	76	4	3	0	0	0
57	3	2	1	1	$1\frac{1}{2}$	77	4	3	1	1	$1\frac{1}{2}$
58	3	2	2	2	3	78	4	3	2	2	3
59	3	2	4	0	$4\frac{1}{2}$	79	4	3	4	0	$4\frac{1}{2}$
60	3	3	0	0	0	80	5	0	0	0	0

If you have occasion for half Pearches, take these Sums twice; if for three quarters take the same Sums thrice, and the total will be your desire.

Note that 3 Barly-Corns is an Inch. 12 Inches a Foot, and 3 Foot a Yard, $5\frac{1}{2}$ Yards a Pearch, 4 Pearches a Day-work, 10 Day-work a Rood, 4 Rood or 40 Pearches an Acre :

This

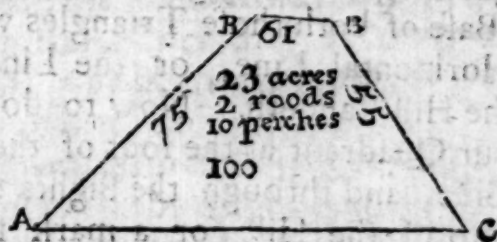
This Table may be very useful for those that have their Chains and Measuring Lines divided only into Pearches, half Pearches, and Quarters: But the most exact way is, to cast up by Chains and Links for such as have Mr. Gunter's Chain, which way you have taught, pag. 231, 232. But I leave every one to use which way pleases him best, and so conclude *Surveying*.

As an Appendix I shall add these 3 Propositions following.

P R O P. I.

How to take the Superficial Contents of Hills and Valleys according to Mr. Diggs.

Suppose ABC were a Mountain, first, measure the Circuit of the Base AC 100 Pearches: Then measure the Compass of the Summity or Top BB 16 pearches,



which added together makes 116 pearches: Then measure the ascent of the Hill AB 75
M 3 pearches,

pearches, and its opposite Ascent CB 55 pearches, adding these two Ascents together 130, half of which is 65, which multiplied by half of the Circuit 58, the Content will be 23 Acres, 2 Roods, and 10 pearches, the true content of the Hill.

For measuring a Valley instead of the Circuit at the Base in a Mountain, you must measure the Circuit of the top of the Valley, and instead of the Circuit of the top of the Mountain, you are to measure the Circuit of the depth of the Valley, and instead of the Ascenses of the Hill, measure the Descenses of the Valley, which makes it an hollow hill turn'd upside down, and to be measured by the Rule aforegoing.

P. R O P. 2.

To find the Horizontal Line of a Hill or Mountain.

This is no more but resolving a Hill into 2 right Angled Triangles, and then to find the Base of both those Triangles which is the Horizontal Line, or the Line on which the Hill standeth. Now to do this, place your Quadrant at the foot of the Hill on one Side, and through the Sights espying the top of the Hill (or a mark set as high above it, as your Instrument stands above the ground) Observe what degrees are cut by the Thread upon the Limb of the

the

the Quadrant (suppose 47 d.) then the Angle at the top must be the Complement thereof to 90, viz. 43 d. then measure the distance betwixt the top and the bottom, which suppose 71 feet, being the Hypotenusal of the Triangle opposite to the right Angle 90, so having these given, you may easily find the Base and the perpendicular also, if required: For as 90 Degrees is to 71 Feet, so is 47 d. to the perpendicular, or 43 to the Base required. Now having got this part of the Horizontal Line, you must get the other part by proceeding after the same manner from the other side of the Hill, taking the Angles, measuring the Ascent, and finding the Base as above, and these two Bases added together makes up the true Horizontal Line of the Hill or Mountain.

P R O P. 3.

To find the Level betwixt any two places, and whether Water may be conveyed from a Spring-head, to any appointed place.

There is an Instrument called a Water-Level whereby this is best effected, though it may be also done by a Quadrant: For the Water-Level it is only a small Pipe of Glass made close at both ends with Wax,

and filled with Water, excepting a very small vacuity, which will shew it self in the middle, when it lies Level; but upon the least leaning either way, you will find it move accordingly. Now this laid in a piece of Wood, having sights at either end, is a fit Instrument for this purpose: This Water-Level placed at any convenient distance from the Spring-head in a right Line towards the place to which the Water is to be conveyed, as 50 or 100 yards, and having two long Staves, cause one of the Staves to be set up at the Spring head, the other let be erected as many yards beyond your Instrument towards the place to which the Water is to be conveyed: These Station Staves erected perpendicular, and your Water-Level precisely horizontal, look through the Sights to the first Staff, causing a Leaf of Paper to be run up and down the Staff till it be espied through the Sights, there observe how many Feet, Inches, or Parts the Paper resteth upon, which note in your Book: Your Water-Level remaining immoveable, go to the other end of it, and causing a paper to be moved up and down the other Staff as before, note how many Feet, Inches, and Parts the Paper resteth upon: Then if the second Staff be more than the first, deduct the less from the greater, and the Remainder will shew
how

how much the second staff is lower than the first, and so by making several Stations you may find your desire at what distance you please.

P R O P. 4.

Another way by the Quadrant is this, First, go to the place whither you would convey your Water, and let any person stand toward the Spring-head any Number of Yards, and observe the top of a Staff in his hand equal to your eye by the Quadrant: Note the Degrees, and so take the Height as before at several other Distances, till you come to the Spring-head it self, and protracting the several Angles of Altitude as you took your Stations and your distance of yards upon the Ground betwixt Station and Station: And in conclusion, you shall have the difference of Height betwixt the Spring-head, and the place whereunto you desire to have your Water conveyed. This is of excellent use to find whether Coals may be saugh'd or drain'd of Water. First, Boaring, or otherwise finding the Depth of the Coal from the surface of the Ground, and subtracting that Height from the height found betwixt the place above ground where the Coals lye, and the place whither you desire to bring your Water: All Coal be-

ing observed to lay true Level, let the surface of the Ground rise or fall as it will, excepting where it is extraordinarily thrown up or down by that which they call an Horse. This therefore may be done either of these ways aforementioned, that is, either by the Water-Level, or Quadrant, only observe, that at every Miles end there ought to be allowed $4\frac{1}{2}$ Inches more than the strait Level for the Currant of the Water, which in a short distance is not considerable.

O F
NAVIGATION.

C H A P. VII.

HAVING now finished what I intend to say concerning *Surveying*, and taking of Heights and Distances at Land, I come now to treat of that other part of *Geometry*, which is, the taking of Heights and Distances at Sea, to wit, *Navigation*: And I hope this Division of *Geometry* into *Surveying* and *Navigation*, and the Definition of them to be taking of Heights and Distances, the one at Land, and the other at Sea; may not by the Candid Reader be thought incongruous or improper, but proper and Genuine to the present purpose for making them plain and obvious to the Capacity of the meanest; which is the principal aim and design of this small Treatise. But to proceed, the whole Science of *Astronomy*

nomny is a great Ornament and Accomplishment to him that professes *Navigation*; but of absolute Necessity *Navigation* must borrow of *Astronomy* the taking of Heights, viz. in these two particulars, (that is) First, The taking of the Height, or the Meridian Altitude of the Sun: And thereby in the second place to find the Height or Elevation of the Pole: Yet this latter, in the same sense that it is called Latitude, may be the finding or taking of a Distance in *Navigation*, being indeed no more but the Distance *North* or *South* from the Equinoctial, or any other supposed parallel; as Longitude is the Distance *East* and *West* from the first Meridian, or any other supposed Meridian: And as the Rhumb leading from one place to another may be called the Distance run upon such a point of the Compass. Now the taking these Heights and Distances is the Sum and Substance of the Art of *Navigation*. And to Treat of these particulars in order, shall be the method of my intended Discourse upon this Subject.

Now these may be more succinctly comprized in these four Terms, being the same which are used by *Normood*, *Hues*, and others. To wit,

1. The

1. The Difference of Latitude.
2. The Difference of Longitude.
3. The Rhumbs.
4. The Distance run upon the said Rhumb.

Any two of these being known or given, the other two may also be found; whence observe, that there is a two-fold finding of each of these. First, As they are *Data*, to be given or known, and so they are found by observation. Secondly, As they are *Quasita*, sought or required, and so they are found by *Trigonometry*, or *Arithmetical Calculation* in the Resolution of a Plain Right Angled Triangle; so that in conclusion you will find that *Navigation* as well as *Surveying* is still no more but the Resolution of a Triangle: But to proceed Methodically: Seeing there must be always two of these given or known, to find the rest: It will be very necessary to shew you how in the first place they may be found as *Data* by Observation.

S E C T.

S E C T. I.

And first for finding the Latitude or the Elevation of the Pole.

P R O B L. 1.

This is done by observing the Meridian Altitude of the Sun or Stars either by an Astrolabe, Cross-staff, or Quadrant.

How to take the Meridian (or any other) Altitude of the Sun or Stars by the Cross-staff by forward Observation, is the same as in the general use of that Instrument in taking of Altitudes, as is taught pag. 212, 213: And how to take the same Altitude by Thread and Plummets, and backward observation, is also taught particularly, pag. 212. §. 4. But the easiest and readiest way to take the Meridian, (or other) Altitude of the Sun is, to do it by the Astrolabe or Quadrant thus, by that which is called backward Observation, and without looking through the sights: Let the Sun shine through that sight which is next the Center, and bring that Beam to fall right upon the hole of the other sight, and the Thread will fall upon the true Altitude in the Quadrant, or in the same manner the

Index

Index in the Astrolabe will cut the Degrees of Altitude.

Now to find the Meridian Altitude of the Sun, (though the Sun do not shine) may be done by knowing the Declination and Latitude.

For if the Declination be North, you must add it to the Complement of the Latitude, (which is always the same with the Height of the Equinoctial ;) and if the Declination be South, you must subtract it from the Complement of the Latitude, which will give you the Meridian Altitude at any time. As suppose at *London* the Elevation of the Pole (or the Latitude) is 52 d. the Complement whereof to 90 d. is 38 d. (which is also the Height of the Equinoctial.) Now the second of *May* the Sun being in 20 d. 42 m. of *Taurus*, his declination Northwards is 117 d. 56'. 21". which added to 38, makes the Meridian Altitude of the Sun to be 55 Degrees, 56 Minutes, and 21 Seconds.

If it be at the time of the Equinoctial, (*viz.* 11th of *March*, or the 13th of *September*) the Height of the Sun or Star (when they are upon the Meridian) subtracted from 90 Degrees, will shew the true Latitude ; but at any other time you must find out the Declination of the Sun or Stars,
and

and if the Declination be Northerly, then subtract it from the Altitude: If Southerly, you must add it to the Altitude; by which Addition or Subtraction you shall have the Height of the Equinoctial above the Horizon, and that being subtracted from 90 Degrees, will shew the true Latitude of the place where you then are.

P R O B L. 2.

To find the Elevation of the Pole by the Globe.

First, Taking the Meridian Altitude of the Sun, and bring the place of the Sun in the *Ecliptick*, or the Star to the *Brazen Meridian*, and so move the *Brazen Meridian* together with the Globe through the Notches it stands in, till the place of the Sun, or the Star be elevated so many Degrees above the Horizon, as the Sun's or Stars Meridian Altitude is. The Globe standing in this position, the Pole will be elevated according to the true Latitude of the place. *Example,*

The 12th of *June* I find the Sun's place in the beginning of *Cancer*, and the Meridian Altitude of the Sun at *London* 62 degrees; bring the first degree of *Cancer* to the Meridian, and elevate the same 62 degrees above the Horizon, so you will find the Pole elevated at *London* 51 Degrees, 30 Minutes:

Minutes: Or by the former Rule, having the Suns Meridian Altitude 62, and finding by the Table of the Suns Declination or otherwise, that the 12th of *June* the Sun hath his greatest Declination, viz. 23 d. 30 m. subtract this from 62 d. rests 38 d. 30 m. for the Height of the Equinoctial; and that again subtracted from 90 d. leaves 51. 30. for the Latitude or Elevation of the Pole.

And seeing the finding of the Latitude by the Sun depends upon knowing his place in the Zodiack, or his Declination; and if you would find the Latitude by the Stars, you must also know their Declinations. Therefore I shall give you two short Rules for the two former, and a short Table for the latter in this place, but must refer the Reader to larger Rules and Tables when I come to speak of them in Astronomy.

P R O B L. 3.

To find the Suns place by these two Verses.

♈	♉	♊	♋	♌	♍
Evil,	attends,	its,	object,	unvail'd,	Vice,
♎	♏	♐	♑	♒	♓
Vain,	Villains,	jest,	into,	a,	Paradise:

These 12 words signifie the 12 months beginning

ginning at *March*, and the 12 signs in order the Sun enters into every Month, and to know what day of the Month he enters: If the first Letter of the word proper to the Month be a Consonant as *Paradise* belonging to *February*, in that Month he enters the 8th day, but if it be a Vowel, (as all the rest are) add so many days to 8, as the Vowel denotes in order of the 5 Vowels
 a e i o u }
 1 2 3 4 5 } which will shew what day of the Month the Sun enters any of the 12 signs, and reckoning that the Sun goes thorough one Degree every day, you may find the Degree the Sun is in, (and by the hours of the day from Noon) the Minutes of that degree.

P R O B L. 4.

To find the Suns Declination by Logarithmes, or the Lines of Artificial Sines.

As Radius or 90 Degrees is to the Suns greatest Declination (which is always 23 Degrees 30 Minutes) So is (the Number of Degrees or Minutes from the next Equinoctial point to the Suns place) to the Suns present Declination: Suppose the Suns place be 0 degrees, or the beginning of *Taurus*, that is, 30 degrees from the next Equinoctial

ctial point of Aries, his Declination will be found to be 11 d. 30 m. for as 90 to 23, 30, so 30 to 11, 30, the like Proportion will hold in any other.

P R O B L. 5.

To find the Declination of some notable Stars, observe this Table.

North		South	
The first Horte in the wain.	57 27	Spica Virginis.	8 55
The third in the wain.	51 5		
Arcturus.	21 43	Heart of Scorpion.	25 30
The Eagle.	7 57	Orions right shoul-	6 16
The Bulls Eye.	15 54	der.	
Castor.	32 30	Orions left foot.	9 10
Pollux.	28 30	The great Dog.	15 66
The Lyons Heart.	13 40	The little Dog.	5 53
These stars have North Declination.		These have South Declinations.	

Thus much for Latitude.

S E C T. 2.

In the next place, for finding the Longitude.

If the Longitude could be as certainly found as the Latitude, it would render *Navigation* perfect ; but notwithstanding that many have pretended thereunto, yet none

none have attained to it in reality as far as I can yet find : *Theaker* in his Planisphere, which he calls a light to the Longitude, tells you he can by that Instrument as certainly do it, as to find the Rising, Setting, and Southing of the Stars, but if he can, he does very ill express it, and I must conclude, that, *aliquid latet, quod non patet*, and perhaps he might keep a reserve to himself on purpose.

The most Rational ways of finding the Longitude which I meet with are these 4, Collected out of *Varenins*, and *Carpenters* Geography.

P R O B L. I.

First, By an Eclipse of the Moon.

By observing how much sooner the eclipse beginneth at a place of known Longitude, (which you may find by an Ephemerides) than at the place where you are; where you may find the time of the Eclipse by observation with the Telescope, and knowing the Latitude at the same time by the Stars, you may find the true hour of the Night. This done, the difference of this time of the Moons entrance into the Eclipse, or the middle or ending of the same at the place where your observation was made, being

being converted into Degrees and Minutes, and either added or subtracted from the hour of the beginning, middle, or end of the Eclipse at the other place of known Longitude found by the Ephemerides: I say the Degrees and Minutes of the difference betwixt the hour at one place, and the hour at the other place added or subtracted from the Degrees and Minutes of the known Longitude, gives you the Longitude of the place required.

P R O B L. 2.

By a Clock, Watch, or Hour-glass.

If you have a very true Watch, and be to sail from a place of known Longitude, when you depart from that place, by an Astrolabe, or otherwise by the Sun or Stars, observe the hour of the day or night, and set your Watch exactly to the same hour, when you have a mind to know what Longitude you are in, observe again by the Astrolabe, or otherwise the exact hour; and if the Watch agree with the observation, there is no difference of Longitude; or what difference betwixt these two you find added or subtracted to or from the Longitude of the place you came from, shews the Longitude of the place you are

now

now at : This way you see depends upon the exact taking the hour at both places, and upon the exactness of the Watch; therefore in this latter respect an Hour-glass is accounted fitter for Sea.

. P R O B L. 3.

Thirdly, by observation of the Difference in the Sun and Moons motion.

First, You must take for granted, that the Motion of the Moon is 48 minutes of an hour slower in 24 Hours, or 360 Degrees than that of the Sun. Secondly, You must suppose, that by Mathematical helps a Man may know in any place the Meridian, (or South point) and also the hour of the day, and at least by an Ephemerides the time of the Moons coming to the South, or to the Meridian. Now suppose at *London* you find (by an Ephemerides calculated for that place) that the Moon comes to the Meridian (on some set day) at four a Clock after Noon: And you being in the *West-Indies* the same day you observe the Moon come to the Meridian at 10 Minutes after four: Then say by the Rule of Proportion, if the difference of the Sun and Moons motion be 48 Minutes of an hour in 360 Degrees, what will it be in 10 Minutes: Or if 48 give
360,

360, what gives 10? The fourth proportional Number will be 75, which is the distance of that place from *London*, from which number the Longitude of *London* being subtracted (20 degrees) and the Remainder (55) again subtracted from 360, the Residue will shew the Longitude (305.)

P R O B L. 4.

By the Distance betwixt the Moon, and some known Star scituate near the Ecliptick.

But this, and several other ways mentioned in *Varenius* as by the Moons place, her entrance into the Ecliptick, & per *Planetas Joviales*, I shall not trouble the Reader with, but refer him to the Author, 418, 420, 421, and pass on. Now the Difference of Latitude, or the difference of Longitude, is so many Degrees, Minutes, or Miles as the Latitude or Longitude of one place differs from the Latitude or Longitude of another place, the Latitude or Longitude being observed or known at both places; as knowing the Latitude of the place from whence you set Sail, and having sailed a certain number of Leagues, there you take an observation of the Latitude as is before-mentioned: Now if both the places be on the North-side the Equinoctial, or

ON

on the south-side the same subtract the lesser latitude from the greater, the remainder is the difference of Latitude: If one of the places be under the Equinoctial, and the other on the North or South side thereof, the difference of Latitude is the Latitude it self. If one of the places be on this side the Equinoctial, and the other on the other side, the Sum of both the Latitudes added together is the difference of Latitude, or the Distance North and South betwixt the two places; and if the Longitude could be as exactly found, it would shew their Distance East and West: However the difference of Longitude is the Distance of any place upon the Line of East and West from the Meridian of any other place proposed, but because this cannot certainly be found, or however not without much difficulty by observation it is seldom given, but required in *Trigonometry*, and found by the Resolution of a Triangle, as shall be shewed hereafter in its due place, and so much for the two former particulars, the Difference of Latitude and Longitude.

S E C T. 3.

In the third place we come to speak of the third considerable in Navigation, viz. the Rhumb.

Which in plain English is the point of the

the Compass upon which the Ship sails, and this is usually one of the *Data's*, or things given, because the Seaman's Compass by which it is directed or steered always shews this. The finding out of that most admirable vertue of the Magnet, being the greatest advantage that ever hapned to *Navigation*: Indeed the difficulty lies in this, that the Needle touched by the Loadstone in several places admits of several Variations, which must be carefully observed and taken notice of.

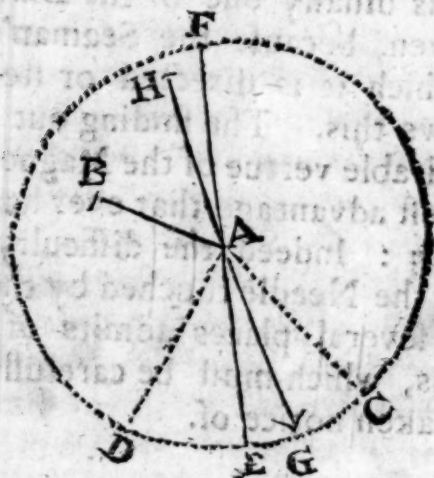
S E C T. 4.

To find this Variation of the Needle or Compass.

There are several ways, as by the *Azimuth Compass*, and by the *Sea-Rings* described by Mr. *Wright*; but these are chargeable. I shall in this place only shew you how to do it these three ways, the one at Land, the other will serve both at Sea and Land.

1. *To find the Variation of the Compass, and withal a true Meridian Line.*

First, Describe several Circles upon a Plain, and in the Center place a Gnomon or Wire perpendicular as AB. In the Fore-
N noon



noon observe when the Extremity of the shadow of AB just touches one of the said Circles, and make there a mark at C. In the Afternoon observe again, when the extremity of the shadow of AB toucheth the same Circle, making there another mark at D. Divide the Distance CD into 2 equal parts, which suppose at E, draw the Line EAF, which is the Meridian Line: Then place a Needle GH upon the Center, and mark how many degrees the point of the Needle G is from E, so much doth the Needle vary from the North in that place.

II. To find the Variation of the Needle by the Globe either at Sea or Land.

First, Rectifie the Globe, and bring the
Suns

Suns place to the East-side of the Hōrizon, and look in the Circle of Winds, and against the place of the Sun you have the point of the Compass upon which the Sun riseth: Then observe by the Compass what point of the Compass the Sun riseth or sets on Morning or Evening, and see what difference there is by the Globe and by the Compass, and that is the Variation (if any be.) Allowing for this variation, the Needle will always shew the Rhumb, on the true point of the Compass upon which the Ship is steer'd.

*III. To find the Suns Amplitude, and there-
by the Variation of the Compass.*

As the Co-sine of the Latitude is to Radius, so is the sine of the Declination to the sine of the Amplitude. Suppose the Latitude $51^{\circ} 32' m.$ its Co-sine or Complement is $38^{\circ} 28' m.$ the Suns Declination is $15^{\circ} 10' m.$ the Amplitude will be found to be $24^{\circ} 52' m.$ North, because the Declination is North. Now the Circumference of the Compass divided into 360° observe at the Suns rising and setting how many degrees the Sun is from the direct point of the Amplitude, and so much doth the Needle vary in that place. This Observation is best to be made, when the lower edge

of the Sun seems just to touch the Horizon.

S E C T. 5.

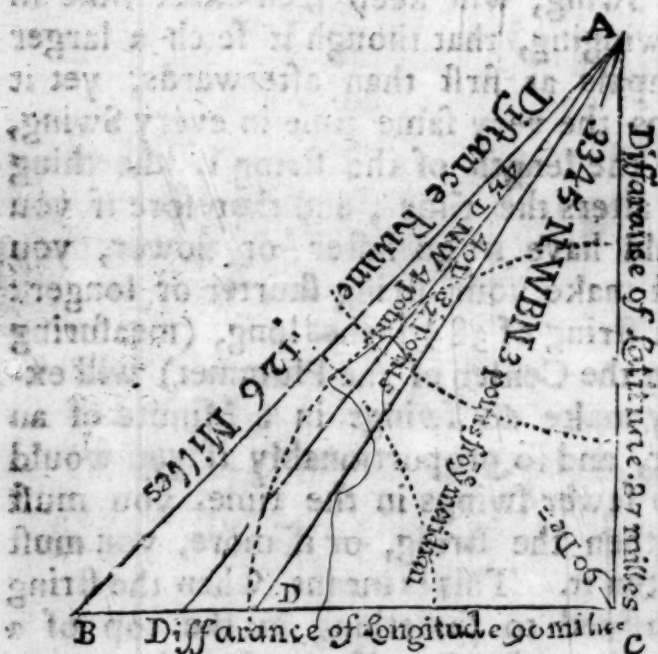
In the fifth and last place, to find the Distance Run.

This is to know what way the Ship makes in such a space of time, or how many Leagues or Miles the Ship runs in an hour, &c. And this is usually observed by the Log-Line and Minute-Glass; so many knots as the Ship runs in half a Minute, so many Miles she saileth in an hour; the space between every knot being $41 \frac{2}{3}$ Feet, reckoning 12 Inches to a Foot, 5 Foot to a Pace, and 1000 Paces to an *English* Mile: But enlarging the Mile according to Mr. Norwood's Observation, the distance between every knot must be 50 Foot, and then as many of those as run out in half a Minute, so many Miles or Minutes the Ship sails in an hour, and for every Foot more the tenth part of a Mile: And if you please, you may bring these Miles into Leagues according to the usual Custom of Seamen; and every time the Log is cast, it ought to be noted upon the Log-board what way the Ship makes. Now besides the Log-line, there hath been several other Engines invented, some with Wheels, and some other

other ways for this purpose; as I have seen a wheel to measure Land withal, and may be applied to know how many Miles a Coach goes in an hour, or the like. And for to measure any small quantity of time, I find no better an Invention than that of a Swing (being proved by Experience) that a Bullet &c. hung in a string, and being moved from the perpendicular to any Angle (keeping the string strait) and so let go to have its own Swing, will keep such exact time in its swinging, that though it fetch a larger Compass at first than afterwards, yet it keeps the very same time in every Swing, only the length of the string is the thing that alters the time; and therefore if you would have it go faster or slower, you must make your string shorter or longer: As a string of $38\frac{1}{2}$ Inches long, (measuring from the Center of the Plummer) will exactly make 60 swings in a Minute of an hour, and so proportionably if you would have fewer swings in the time, you must lengthen the string, or if more, you must shorten it. This is meant when the string is fastned to something in the top of a Room, or so: But if you hold it in your hand, it is observed that $31\frac{1}{2}$ Inches in length will do the same as $38\frac{1}{2}$ fastned to any steady thing. This may very properly be used instead of the half-Minute-glass aboard a Ship.

S E C T. 6.

Thus you see how you may find by observation the Latitude and Longitude, the Rhumb, and the Distance run: any two of which being known (by reason of the Right Angle, which is also always known:). The other two may be easily found by *Trigonometry*, or the Resolution of a Plain Right



Angled Triangle, as you may see in the Triangle ABC: The Hypothenufe always representing the Distance run AB, the Angle at the North point A represents the Rhumb

Rhumb which (in the Triangle DAC is North-West and by North :) in the Triangle BAC is North-West : The Base BC is the difference of Longitude or Departure from the Meridian ; the perpendicular AC is the Difference of Latitude, and the Angle at C you see is a Right Angle ; and the Angle at B should be the complement of the Rhumb.

Now the Transmutation of the things to be granted or known, and to be enquired after in these four Terms, (*viz* the Difference of Latitude, the difference of Longitude, the Rhumb, and the Distance run) may be proposed six manner of ways, and every Proposition resolved by the Resolution of this plain Right-Angled Triangle, as follows in Sailing by the *Plain Chart*.

P R O P. 1.

The Difference of Latitude AC and Difference of Longitude BC being known, The Rhumb and Distance, *i. e.* the Angle A, and the Hypothenuse AB may be found.

This is the same as having two sides and a Right Angle berwixt them, to find the other two Angles, and the third side, as is taught *pag.* 137. Or by Protraction, thus,

N 4

First,

First, lay down the Base 90 Miles, being the difference of Longitude BC, and upon the point C protract an Angle of 90 *d.* laying down the perpendicular 87 AC, being the difference of Latitude, and to touch the ends or extremities of both these draw the Hypothenuſe or Distance run 126 AB, ſo ſhall the Angle at A be the Rhumb 45 *d.* *vide pag. 132.*

P R O P. 2.

The Difference of Longitude BC, and the Rhumb A being known; the difference of Latitude, ſide AC, and the Distance run, *s.* AB may be found.

This is the ſame, as to have 2 Angles and a ſide given, to find the other two ſides: which is done by the general Rule, *pag. 141.* For as the Angle at A, which represents the Rhumb 45 is to the Baſe BC 90 Miles, ſo is the Right Angle at C 90 *d.* to the Distance *s.* AB 126 Miles, and ſo is the Complement of the Rhumb, *viz.* 45 at the Angle B to the perpendicular or difference of Latitude 87 Miles AC, either to be wrought by Logarithmes, *pag. 31.* or by the Lines of Numbers, Sines, and Tangents,

gents, pag. 41. or by Protraction, pag. 132. to which places I refer the Reader.

P R O P. 3.

The difference of Longitude BC, and the Distance run s. AB, being known.

The Difference of Latitude s. AC, and the Rhumb Angle A may be found.

Supposing the Right Angle at C 90 d. to be known with the other 2 sides BC and AB. This is the same, as having two sides and an Angle opposite to one of them, to find the other Angle opposite to the other side given, and the third side. Therefore by the Rule of 3 say, as 90, or Radius is to s. AB 126, so is the s. BC 90 Miles to the Angle A 45 d.

P R O P. 4.

The Difference of the Latitude AC, and the Rhumb, the Angle at A being known.

The difference of Longitude BC, and the Distance run s. AB. may be found.

In this Proposition (supposing the Right Angle also given) you have all the three Angles given, the third Angle being the

N 5

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Complement of the Rhumb to 90, viz. 54^d. Therefore say by the Rule of Proportion, As the Complement of the Angle at A, viz. the Angle B 45 ^d. is to the *s* AC, so is the Rhumb or Angle at A 45 ^d. to the *s*. BC, and so is Radius or 90 ^d. to the *s*. AB.

P R O P. 5.

The Difference of the Latitude AC, and the Distance run AB, with the Right Angle C being known,

The Rhumb A, and Difference of Longitude BC, may be found.

This is to have two sides and an Angle opposite to one of them given, to find the Rest, as in Prop. 3. As the Angle C is to *s*. AB, so is *s*. AC to the Angle B, and so is its Complement A to *s*. BC, which Complement is the Rhumb, and the side BC is the difference of Longitude required.

P R O P. 6.

The Rhumb, Angle at A, and the Distance run AB, with the Right Angle being known,

The Difference of Longitude BC, and Difference of Latitude AC may be found.

This

This Proposition is of all other the most useful, these being the most usual *Data's* to find the other; most Seamen keeping an Account only of the Rhumb or Point of the Compass sail'd upon; and the Distance run by often casting the Log-line.

And it is the same as having 2 Angles and a side opposite to one of them given, to find the rest of the Triangle, as *Prop. 2, 3, 5*, therefore by the Rule of Proportion, say, As the Angle C 90 d. is to s. AB the distance run, so is the Rhumb or Angle A to the s. BC the Difference of Longitude, and so is the Complement of the Rhumb to 90, viz. Angle B to s. AC the difference of Latitude. In this Proposition, as also in any other, where the Rhumb is given, as *Prop. 2, 4*, having the Rhumb given, and the Right Angle, you have also the third Angle given, being the Complement of the Rhumb to 90, which makes it very easie to be resolv'd. And thus you see these 6 Propositions of Plain Sailing, (or if there can be more Transformations of these 4 principal Terms in Navigation) may all be resolv'd by Trigonometry: So that hence you will clearly discern, that not only Surveying, but Navigation also is performed by the Resolution of a Triangle, and that only a Plain Triangle; and Navigation (altogether) and Surveying for the most part, by a Right

Right Angled Plain Triangle only : whereby the easiness of both, and the excellency of *Trigonometry* do most manifestly appear.

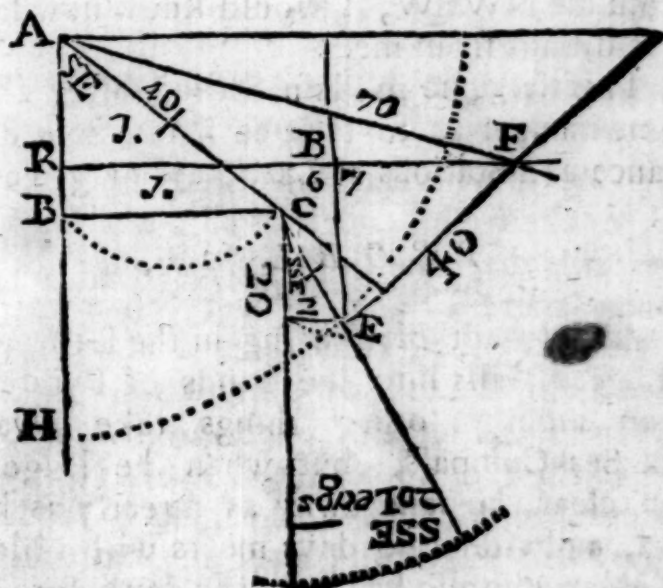
S E C T. 6.

A Travis.

What hath hitherto been spoken concerning *Navigation*, respects only a direct single Course where a Man needs not Lave, or change his course at all ; and for a *Travis* it is the same thing only wrought by so many more Triangles, every time you change your course constitutes a new Triangle, as by the following Example.

As suppose, I set out from *Sommars Island*, lying in the Latitude of 32 d. 20 m. North Latitude, and Sail SE 40 Leagues ; from thence I Sail SSE, till I alter my Latitude one Degree ; and from thence upon some point between North and East I Sail 40 Leagues, till I alter my Latitude one Degree, 45 m. I demand what Latitude I am in ; my distance from the place whence I set Sail : My departure from my first Meridian, likewise my Course made good : When all this is done and protracted, the difference of Latitude is AR, reckoning 20 Leagues, or 60 Miles to a Degree, is 21 Leagues, or 63 Miles ; and therefore in degrees

grees, 1 d. 3 m. which because it is Southerly, subtract from your first Latitude, (if it had been Northerly, you must have added it thereunto.) And that is your present Latitude.



Secondly, The distance run (as if you had gone directly streight) is AF 70 Leagues. taken from the same Scale of equal parts, as AC or EF was taken.

Thirdly, The Departure from the Meridian is RF 67.

Fourthly, The point of the Compass that the Ship lies now from *Sommars Island*, is from H to F, viz. ESE half a point Easterly.

P R O B L

P R O B L. 1.

Sailing away WSW, I see a point of Land which I find to bear from me WBN, and having Sailed 6 Leagues further I find it bears from me NWBW, I would know how far it is distant from me.

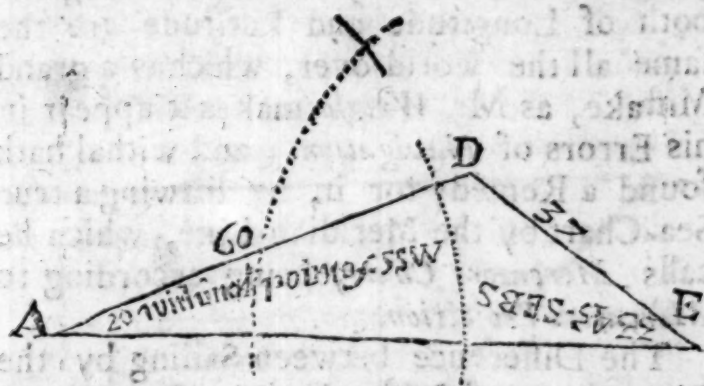
This is done in Plain Sailing after the same manner as to take an inaccessible distance at 2 Stations at Land, as in pag. 197.

P R O B L. 2.

A Merchant Man being in the Latitude of 43 d. falls into the hands of Pyrates, who amongst other things take away his Sea-Compass, but when he is gotten clear, he sails away as directly as he can, and after two days meets with a Man of War, who also had been the day before in the Latitude of 43 d. and had Sailed thence SEBS, 37 Leagues. He being desirous to find those Pyrates, the Merchant Man tells him he left them lying to and from where they took him, and he had Sailed since at least 64 Leagues between the South and West, what Course shall the Man of War shape to find these Pirates.

This is done much after the same manner, as the former, suppose AE the Parallel of 43 d.

43 d. D the place where the Ships meet, the Angle at E, the Rhumb upon which the



Man of War Sailed SEBS 33 d. 45 m. DE his distance 37 Leagues, AD the distance Sail'd by the Merchant Man. Now by these you are to find the Angle A, which you will find 20 d. within $\frac{1}{4}$ point of SSW, so hath the Merchant Man sailed, and therefore the Man of War must Sail NNE, within $\frac{1}{4}$ point of the Compass.

I might add many more such like as you find them in *Norwoods* Epitomy, to which I refer the Reader; done after the manner of Plain Sailing, supposing the Degrees of Longitude and Latitude are both equal.

This *Travis*, as also the eight former Propositions are all wrought by that way of Sailing, which we call *Plain Sailing*, or Sailing by the Plain Chart, which is very easie and also exact enough for short Distances, and near the Equinoctial; in other Voyages

it is subject to Error, as supposing the Degrees of Longitude to be the same in every parallel of Latitude; or that the Degrees both of Longitude and Latitude are the same all the world over, which is a grand Mistake, as Mr. *Wright* makes it appear in his *Errors of Navigation*, and withal hath found a Remedy for it, by drawing a true Sea-Chart by the Meridian-Line, which he calls *Mercators Chart*, being according to *Mercators Projection*.

The Difference between Sailing by the *Plain Chart*, and Sailing by *Mercators Chart*; that is, the Difference betwixt keeping an account either by the one, or by the other, consists in these particulars.

First, In the *Plain Chart* both the Meridians and Parallels, or (in plain Terms) the Degrees of Longitude and Latitude, are all equal, and measured by the same Scale of equal Parts, and therefore by consequence the Distance must be also measured by the same Scale, and so all the three sides of any Plain Right Angled Triangle representing these, *viz.* The Base, the Perpendicular, and the Hypothenuse are all measured by one and the same Line of equal Parts.

But finding this way, *viz.* by the *Plain-Chart* very erroneous, being grounded upon
errone-

erroneous principles, as is shewed by Mr. *Wright* in his errors of *Navigation*, as also by *Leyburn* in his ninth Geometrical Exercise. The Projection of the *Mercators Chart*, and keeping a reckoning at Sea by that, discovers the true Longitude, Latitude, and Distance most exactly. For the Meridians being great Circles of the Globe are always equal, and the Degrees of Latitude measured thereupon are also equal:

But the Parallels (to the Equinoctial) are all lesser Circles of the Globe, and towards either Pole always grow lesser and lesser, (as may most evidently appear by the Globe it self if you please to cast your eye upon it :) And therefore the Degrees of Longitude which are measured upon these Parallels do differ in every Latitude: Therefore one side of *Mercators Chart* is the Meridian Line of unequal Parts, and the other side is a Line or Scale of equal Parts only: Upon the Meridian Line of the *Chart* is measured the Difference of Latitudes, and the Distance upon the Rhumb. The Difference of Longitude is measured upon the Equinoctial Line, or Line of equal Parts: And in *Trigonometry*, because the sides of all *Triangles* ought to be brought to the same proportional parts one with another, that is the meaning of Meridional

nal Parts; for instead of the unequal Parts of the Meridian Line, you take the equal parts of the Line of Lines answering thereunto: Now how to find these Meridional Parts in reference to the Latitude you have plainly set down, *pag.* 283. The same Rules will also serve for the Distance between any two places, provided that taking the same Distance in your Compasses, you set one Foot of your Compasses as much below the lesser Latitude, as the other above the greater Latitude.

Then by the former Rules, *pag.* 283, find the Meridional Parts answering to the Degrees on the Meridian Line intercepted betwixt the feet of the Compasses, and those are the Meridional Parts answering to the Distance upon the Rhumb, and to be used instead thereof.

So that you see in short, the difference betwixt Sailing by the *Plain Chart*, and *Mercators Chart*, which differ in this, and this only, That in the *Plain Chart* both the difference of Longitude and Latitude, as also the distance are all measured by the same Scale of equal parts; but in *Mercators Chart* the difference of Longitude only is measured upon the scale of equal parts, and the difference of Latitude & distance must be always measured upon, and taken out of the Meridian Line of the Chart: And when either the difference
of

of Latitude or the distance is opposed in Proportion to the Longitude, instead of the Degrees of the Meridian Line you must use the Meridional Parts answering thereunto, which by most are called the Meridional Distance, or Meridional difference of Latitude; but by Mr. *Leyburn* in his *Geometrical Exercises*, pag. 18. They are called the proper distance, and the proper difference, which I shall retain in some of these following Problems taken out of the same Author.

S E C T. 7.

In Gunters Sector the Meridional Line being placed on the same side with the Line of Lines, you may thereby easily find these Meridional Parts by these Rules.

1. If you would know the difference of Latitude in Meridional parts betwixt two places, the one lying under the Equinoctial, (as the River of *Amazones*) the other in 50 degrees of North Latitude, suppose (the Lizard). Look for 50 on the Meridian Line, and right against it on the Line of Lines you find 57 d. 54 m. or in decimal parts of a degree, 57 d. $\frac{2}{3}$, and these are the Meridional Parts required.

2. If both the places have Northerly or Southerly Latitude, as suppose one in 30 d. the other in 50 d. Extend your Compasses from

from one of the Latitudes to the other, as from 30 *d.* to 50 on the Meridian-Line. The same Extent will reach from the beginning of the Line of Lines to 26 *d.* 26 *m.* which is the Meridional parts required.

3. When one of the places hath South-Latitude, and the other North, as suppose 10 *d.* South, and the other 30 North. The Extent from the beginning of the Meridian Line to 10 degrees shall reach from 30 *d.* on the Meridian Line, to 41 *d.* 31 *m.* on the Line of Lines or equal parts, which are the Meridional Parts required.

Knowing then how to find these Meridional parts, note that the difference of Latitude in these is called the Meridional difference of Latitude, and the true difference of Latitude is that which is found by subtracting the lesser Latitude out of the greater, the Remainder is the true Latitude. This being premised, I shall now proceed to these several Propositions.

P R O B L. 1.

The difference of Latitude, and difference of Longitude being given or known, to find the Rhumb.

As the difference in Latitude is to the difference

difference in Longitude, so is the Tangent of 45 to the Tangent of the Rhumb.

Suppose the two places be the Lyzard in 50 degrees North, the other *St. Christophers* in 15 d. 30 m. the proper difference of Latitude is $42 \frac{1}{10} \frac{2}{10}$, and the difference of Longitude 68 d. 50 m. Therefore the Extent on the Line of Numbers from $42 \frac{1}{10} \frac{2}{10}$ to 68 d. 50 m. shall reach the same way from the Tangent of 45, to the Tangent of 58 d. 26 m. which is the Rhumb leading betwixt those 2 places.

P R O B L. 2.

The Difference of Latitude and the Rhumb given, to find the difference of Longitude.

As Radius, or the Tangent of 45 to the Tangent of the Rhumb, so is the proper difference of the Latitude to the difference of Longitude.

P R O B L. 3.

By one Latitude, the difference of Longitude and the Rhumb given, to find the other Latitude.

As the Radius is to the Co-Tangent of the Rhumb from the Meridian, so is the
disse-

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difference of Longitude to the proper difference of Latitude.

P R O B L. 4.

Having the Latitude of one place (50 d.) and the Rhumb (33 d. 45 m.) leading from that place to another unknown; and the distance upon the Rhumb from the first place to the second, (6 d.) To find the difference of Longitude (5½ d.)

As the Radius is to the sine of the Rhumb from the Meridian, so is the proper distance upon the Rhumb to the difference of Longitude.

P R O B L. 5.

The difference of Longitude, one Latitude and the Rhumb given, to find the distance.

As the Sine of the Rhumb is to the difference of Longitude, so is the Radius to the proper distance.

P R O B L. 6.

Difference of Longitude, and distance with the Latitude of one of the places given, to find the Rhumb that leads from one to the other.

As

As the proper distance is to the difference of Longitude, so is Radius to the Sine of the Rhumb.

P R O B L. 7.

The Longitude and Latitude of two places being given, to find the distance and Rhumb.

As the proper difference of Latitude is to the Radius, so is the difference of Longitude to the Tangent of the Rhumb from the Meridian: And as the Sine of the Rhumb is to the difference of Longitude, so is the Radius to the proper distance.

P R O B L. 8.

The difference of Latitudes and their distance upon the Rhumb given, to find the difference of Longitude.

As the proper distance is to Radius, so is the proper difference of Latitudes to the Co-sine of the Rhumb from the Meridian: And so is the Sine of the Rhumb from the Meridian to the difference of Longitude.

P R O B L. 9.

The Difference of Longitude of two places, their Distance upon the Rhumb, and the
Latitude

Latitude of one of the places being given, to find the Difference of Latitudes.

As the proper distance is to the Radius, so is the difference of Longitude to the inclination of the Rhumb to the Meridian: And so is Co-sine of the Rhumb to the difference of Latitudes.

P R O B L. 10.

By one Latitude, Distance and Rhumb, to find the other Latitude.

As Sine 90, or Radius is to the Cosine of the Rhumb, so is the distance to the true difference of Latitude.

I shall now proceed to shew you Mr. Philips's way of drawing a *Mercators Chart*, and how to measure both Longitude, Latitude, and Distance thereupon, and to keep an account of a Voyage thereby, as is performed by him in his *Geometrical Seaman*, which I recommend as a Book of very excellent use to Seamen, and such as desire to be more at large informed in all the particulars concerning *Navigation*, in all the three kinds: Of Sailing by the *Plain Chart*, *Mercators Chart*, or by the *Arch* of a great *Circle*. I shall in this place content my self only to Epitomize the Use and Figure of his *Mercators Chart*, as follows.

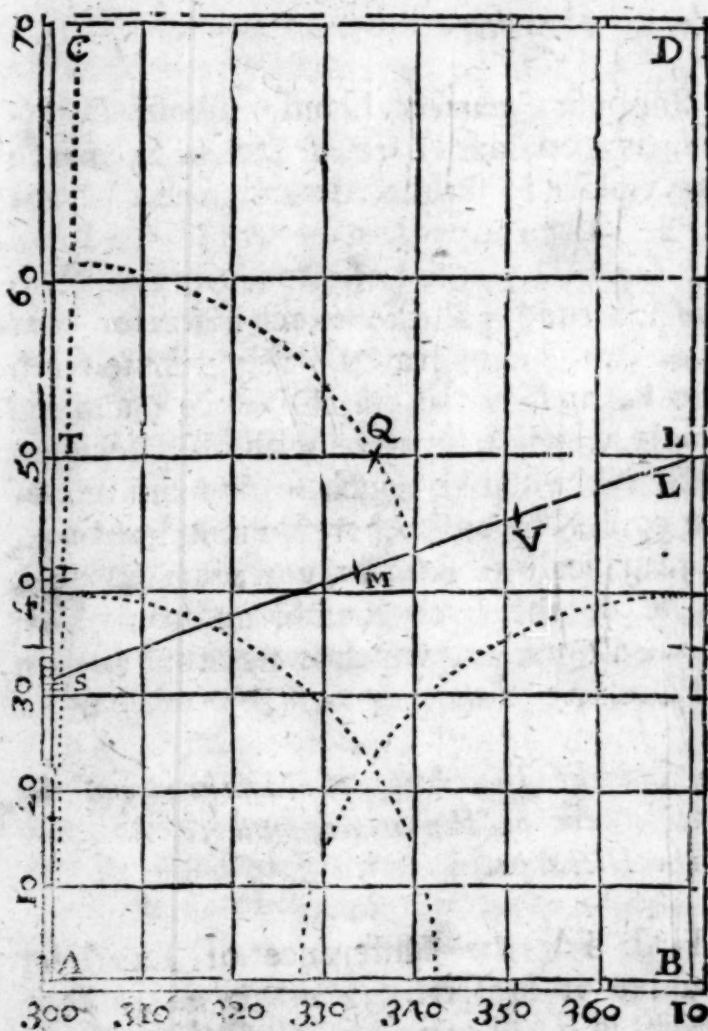
Knowing the Longitude and Latitude of two places, to set them upon the Chart, and to find the Rhumb leading from one of the places to the other.

Suppose *Summers Island*, whose *Longitude* is 300, and *Latitude* 32 d. 25 m. the exact place to set that down in the Chart is at S. Then suppose you would set down the *Lyzard*, whose *Longitude* suppose 10 d. and *Latitude* 50, the exact place for that is at L. Then draw a streight Line from S to L, and that is the Distance; and the Angle which it makes with the Meridian of S, is the Rumb which leads from one of the places to the other, which measured in your Scale of Rumbs, you will find 71 d. 21 m. or the sixth Rumb, and something above a Quarter, which is ENE more than $\frac{1}{4}$ Easterly.

To find the Difference of Latitude, and the difference of Longitude, and to Measure the Distance.

First, For the Difference of Latitude, you see it is from 32 d. 30 m. to 50 d. which is 17 d. 30 m. The difference of Longitude is 70 d. To measure the distance upon the Rumb betwixt S and L, do thus, First, Divide the distance that is in

the Meridian Line between the two Latitudes into two equal parts, which in this Example will fall at 42 Degrees. Then



The Equinoctial Line.

take with your Compasses half the length
of

of the Line LS which is LM, and with that distance set one foot of your Compasses in 42, the middle point betwixt the two Latitudes, you will find the other foot will reach downwards to 8 d. 30 m. and upwards to 63 d. 30 m. The Degrees in the Meridian Line betwixt these 2 points are 55 degrees, which is 1100 Leagues, and this is the true distance upon the Rhumb betwixt S and L.

To Measure a Parallel Distance between two places.

Suppose the places to be L and T both in the Latitude of 50 degrees, their difference of Longitude being 70, divide the distance betwixt L and T into 2 equal parts which is at Q. Take LQ in your Compasses, and setting one foot in 50 Degrees of the Meridian Line, or the degrees of Latitude, the other foot will reach downwards to 22 d. 30 m. and upwards to 67 d. 30 m. and if you count the degrees intercepted between these two points, you will find them 45 d. which multiplied by 20, gives the distance between these two places 900 Leagues.

If two places differ only in Latitude and lye under the same Meridian: The Degrees between them upon the Meridian Line gives the Degrees, which multiplied

by 20 bring them into Leagues, or by 60 into *English Miles*.

Having Sailed from the *Lyzard* about the Distance of 15 degrees upon the same Rumb as in the Chart, you would know what Longitude and Latitude you are come to, or the point where the ship then is.

Take 15 degrees in your Compasses from 50 (or L, which signifies the *Lyzard*) out of the Meridian line; or the degrees of latitude whatsoever they be, which will reach to 35 d. in this Example.

Set this distance upon the Rumb-line you Sail upon, which will be at V; lay your Ruler by that mark from 46 and 46 in the Meridian-line, and that is your latitude; do the like for your longitude, and you will find it 351, or thereabouts.

To avoid Sailing in a Parallel that is full East and West at any time.

It will be your best way to steer no nearer to the East or West than the 7 Rumb from the Meridian, for so you may by your observation of the latitude correct your account, which otherwise you cannot do, and in doing thus you will not go much out of your way; for if the two places be distant 10 degrees, if you sail one half of your way upon the seventh Rumb, you will raise the Pole

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BOOK II. Of Navigation.

293

Pole very near 1 degree, viz. $1^{\circ} 00' 04''$, and if you lay the Pole as much upon the seventh Rumb the other way, so you will come to the place desired, having run the same distance as before, viz. 5 leagues. Thus in a Voyage of 10 degrees, or 200 leagues, you go not fully 4 leagues out of your way; which is not one in 50, which will be well recompensed, because in Sailing thus upon the seventh Rumb, you may by the observation of the latitude correct your Account.

Note that a Travis may be drawn upon this Chart, as that in pag. 277, is done upon the Plain Chart, *mutatis mutandis* in the latitude and Distance.

And thus much shall suffice to have spoken of Sailing by the *Plain Chart*, or *Mercators Chart*. Those that have a mind to understand the third kind of Sailing by the Arch of a great Circle, they may consult *Philips* his Geometrical Seaman, and other Authors: I shall not here insist upon it, conceiving it to be rather a piece of Nicety, than any great matter of solidity which I discern in it; yet withal do take it for an unquestionable verity, that it is the nearest way by reason of the Spherical Form of the Earth and Sea, and circularity of the Terrestrial Globe. But to let this pass, I shall conclude what I intend to say concerning Na-

vigation in these following particulars.

1. To shew how many leagues do answer to one degree of longitude in every several latitude.

2. To shew how many leagues do answer to a degree of latitude in every several Rhumb.

3. To know the Burden of a Ship, and how to Build a Ship to any proportion or bigness.

1. To know how many leagues do answer to a degree of longitude in every several latitude.

As the Radius (or sine of 90 *d.*) is to 20 in the Line of Numbers, (or 60 if you desire Miles) so is the Cosine of latitude (at *London* 38 *d.* 38 *m.*) to the number of leagues or Miles in that latitude (*viz.* at *London*) 37 $\frac{1}{10}$ Miles, or 12 $\frac{1}{2}$ leagues, *fere.*

To do this by the Sector,

Take 20 leagues out of the line of lines, and make it a Parallel Radius by fitting it over in the sines of 90 and 90, so this Parallel sine taken out of the complement of the latitude, (*viz.* 25. 15.) will give the leagues (*viz.* 18.)

To do this another way :

Suppose you have a Quadrant, the one side whereof being the full Radius may be divided into 20 equal parts, the other side into

60; then with your Compasses take the nearest distance from the complement of the latitude to the side of the Quadrant, and that distance measured upon the line divided into 20 from the Center, will give the leagues upon the line of 60, which will give the miles answerable to one Degree of longitude in that latitude. Other ways there may be for doing this, but these are sufficient.

2. To know how many leagues do answer to a degree of latitude in every several Rhumb, or more Plainly to know how many leagues you are to sail upon each Rhumb before you raise or depress the Pole one Degree.

As the complement of the Rhumb is to 20 leagues the measure of one degree at the Meridian, so is the Radius to the leagues answering to one degree upon the Rhumb.

As suppose Sailing NEBN from 50 *d.* of latitude, it were required how many leagues the Ship should run before it should come to 51 *d.* of Latitude: Because this is the third. Rhumb, and the Inclination thereof 33 *d.* 45 *m.* The Complement thereof is 56 *d.* 15 *m.* Therefore the Extent of the Compasses from 56 *d.* 15 *m.* upon the line of Sines to 20 upon the line of Numbers, the same extent the same way will reach from Radius or the Sine of 90 *d.* to 24, so many leagues must be sailed to raise the Pole 1 *d.*

To do this by the Sector.

Take 20 leagues from the line of lines, and make it a parallel line of 56 d. 15 m. and his parallel Radius between 90 and 90 measured in the same line of lines, will shew you 24 for the Number of leagues required.

Sailing with the Ship.	North or South	You keep the same Meridian			And 20 English leagues	Do either Raise or Depress the Pole one Degree.
		Hav. changed your				
	Upon the 1st Rumb	Parallel a degree of			& 20 $\frac{1}{3}$	
		Latitude, your de- part. from the Me- ridian is 4 Leagues.				
	Upon the 2	Rumb.	8 $\frac{1}{3}$	Leagues.	& 24 $\frac{2}{3}$	
	3		13 $\frac{1}{3}$		& 24	
	4		20		& 28 $\frac{1}{3}$	
	5		30		& 36	
	6		48		& 52	
	7		101		& 102 $\frac{2}{3}$	
East or West	You are still in a Parallel, and neither raise, nor lay the Pole at all.					

To Convert Degrees and Minutes into Hours and Minutes.

Note that 15 Degrees makes an Hour, therefore divide the degrees by 15, the Quotient will be Hours: The Fraction (if any

any remain) multiply by 4, (because every Degree is 4 Minutes of time) and to the Product add every 15 Minute of a Degree (if any be) for so many makes a Minute in time. This will exactly reduce Degrees and Minutes to Hours and Minutes. As for Example 118 d. by 15. the Quotient is 7 Fraction 13, which multiplyed by 4, is 52. Total 7 H. 52 M. as above.

But for your ready finding of your true Northing and Southing, or Easting and Westing, or in other Terms your Difference of Latitude and difference of Longitude by the Rumb and distance given, or by knowing how many Leagues you have sailed upon any Rumb: You have excellent Tables in *Philips's Geometrical Seaman*, and in *Sturmy*, or to supply those you may have a Traverse Scale, which is a Sliding Scale, on one side having a Line of Numbers, on the other side the Rumbs: The use of it is thus; Set your distance run against the Radius (which is the eighth Rumb) and the Number of Leagues against the Rumb you sailed upon is the difference of Longitude, or your departure from the Meridian, or your Easting or Westing (call it which you will) and the Number of Leagues opposite to the Complement of the Rumb is the difference of Latitude, or your Northing or Southing: As for Exam-

ple, suppose you Sail 57 Leagues upon the third Rhumb, and would know how many Leagues you have altered your Longitude or Latitude: Set 57 the Leagues sailed to Radius (or the eighth Rhumb;) look against the third Rhumb, there you find $31 \frac{2}{10}$ leagues for your departure from the Meridian or difference of Longitude, and against the complement of the said Rhumb, which is 5, or the fifth Rhumb, there you will find $47 \frac{4}{10}$ leagues, and so many leagues is your difference of Latitude: The like of all other.

The Use of the Sea Quadrant.

This Instrument consists of 3 Vanes and 2 Arches, one of 60, and the other 30, which together make 90 Degrees, and therefore call'd a Quadrant: The 3 Vanes are the sight Vane upon the 30 Arch, the Shadow Vane upon the 60 Arch, and the Horizon Vane at the Angle A: So turn your back of the Sun, and move the Sights, till the shadow and the Horizon be seen together, and then upon the 60 or lesser Arch observe the Degrees cut by the upper edge of your Shadow-Vane, and add those to the degrees cut by the inside of the Sight-Vane upon the 30 Arch, which Degrees added together, the Sum is the Complement of the Altitude, or the Suns distance from the Zenith.

To

To keep a reckoning upon a Plat of a whole Voyage.

Suppose you Sail from the *Lizard* South-Eastward (which we suppose in the Latitude 50, 00 degrees) and the difference of latitude 3 degrees, which deduct from 50 d. and then the latitude the Ship is now in is 47d. and suppose your course run 50 leagues Eastward.

Therefore to set the place of your ship upon your Plat, you must use two pair of Compasses, with one pair take the extent between the latitude 50 degrees and 47 degrees, and with that extent set one point of your Compasses in the *Lizard*, the other extend towards the place where the Ship is now, but so that your Compasses stand parallel to a North and South Line. This done, keep one foot of your Compasses in that point, and with the other pair take the departure 50 leagues from the Scale of leagues, then interchange your Compasses, placing this last pair in the point where the other pair stood, set this departure 50 leagues to the Eastward; but so that your Compasses may stand parallel, and to an East and West-Line. (This you may find by trying with your other Compasses whether the legs be æquidistant from the next East and West-line.) / This

This second point thus found upon the Plat is the place of the Ship (according to your reckoning) which was required. This I have seen practised by several Ship-Masters upon Maps, and is a very ready way, marking the place where the Ship is with Chalk, which may be rubbed out again when you come at the next Station.

How to know what Burthen a Ship is of.

The general receiv'd Opinion is to take the length at the Keel, the Breadth at the Beam, and his Depth in Hold, which multiply one in the other, and divide the last product by 100, gives you his Tunnage, which is the Kings allowance: But Merchants divide by 110; but you must note this is of such Ships as are built Taper-wise from the Keel, but others think 95 a number reasonable.

Length 75 foot	} Product is 28275, which	
Breadth 29		divided by 100, gives 282 $\frac{3}{4}$
Depth 13		Tuns.

Supposing a Ship of 100 Tun, and you would make another Ship of double Burthen.

Suppose this Ship of 100 Tun be 44 foot long by the Keel, 20 foot broad midship Beam,

Beam, 9 foot deep in Hold. Multiply each of these Numbers cubically ; double the Product, and then extract the Cube Root. As for Example, Multiply 44 by 44, makes 1936, and that again by 44 makes 85184 for the Cube number, which being doubled (because the Ship is to be double the burthen of the other) so it makes 170368: Extract the Cubique Root of this Number, and it will yield 55 feet $1\frac{41}{100}$ parts, which is 5 Inches, and almost $\frac{1}{4}$ Inch, and so you must do by all the other dimensions of the Ship to find them, and so make them proportionable.

What due Proportion ought to be used in the Building of all Ships whatsoever.

The due proportion of a Ship is, That the longitude of the Vessel whatsoever it be more or less ought to be divided into 300 equal parts ; of the which parts 30 must be assigned to the Depth, and the Breadth shall contain 50, or the $\frac{1}{6}$ part of the longitude, so shall the Ship be both proportionable, and more safe for Traffick.

To conclude pleasantly, I shall here give you Bishop *Wilkins's* device in his *Mathematical Magick* of making a Ship to Sail under Water : The Difficulties herein he resolves thus.

1. No

1. No Land Creatures (especially Men) can live without Air wherein they may breathe, and therefore such a Vessel must be very large and spacious ; so as to allow Refrigeration.

2. If so, How is it possible that they should swim or sail along ? To this he says, That those Bodies which are carried on the Water, be they never so big or ponderous, (suppose equal to a City or whole Island) yet they will always swim on the top, if they be but any thing lighter than so much Water as is equal to them in bigness.

3. How to poise this Vessel so equally, That it shall neither rise too high, nor sink too low : This he propounds to be done by hanging more or less great weights at the bottom.

4. How the motion of this Vessel must be made, which must be by Oars put thorough the sides of the Ships with leather-bags made fast to the Hole, and tyed close to the Oars, by which means the Oars may move, and yet let in no Water at all.

5. The same device doth remedy the fifth and last, and greatest difficulty, which is to take any thing into the Vessel, or put any thing out of the Vessel without admission of any Water : How this may be done, he propounds thus ; Suppose a leather-bag of what bigness you please made fast
and

and close to a Window round about, being also ty'd close about towards the Window, then any thing that is to be sent out may be safely put into that end within the Ship, which being again close shut or tyed, and the other end loos'd, the thing may be safely sent out : And by the like means how to receive any thing into the Ship or Vessel is easie to conceive.

But I shall sail no further in this Discourse, the Voyage in such a Vessel being near as difficult and dangerous to perform, as to fly into his *New World of the Moon*, though he tells you *pag.* 219. That if a Body were above the Sphere of the Magnetical Vertue of the Earth (which he supposes 20 Miles upwards, or so) he might there stand as firmly in the open Air, as he can now upon the Ground.

Or Secondly, If this Sailing under Water could be performed, it would not be of much usefulness; therefore shall rank this amongst those other conceits which are demonstrable, but not practicable, such as pulling up Trees by the Roots with a single hair, or by the breath of a mans mouth : Or *Pythagoras* writing in the Moon by reflection : Or *Archimedes* his moving *datum pondus cum data potentia* ; that if he did but know where to stand and fasten his Instrument, he could move the whole Terrestrial Globe.

Globe. More useful (if at all practicable) is his perpetual motion ; but both useful and practicable, and in many places practic'd and us'd is his Sailing Chariot, like that in *Holland*, which with the Wind would pass from *Scheveling* to *Putten* in two hours, being distant more than 42 *English* Miles.

But paying due Honour and Respect to the mention of this Reverend, Learned, and Ingenious Author, I shall here conclude what I have to say of *Geometry* in general, and in particular as to *Surveying* and *Navigation* ; and proceed according to my propounded Method to more solid and serious *Mathematical Propositions* and *Practical Conclusions* in *Astronomy*.

The Third B O O K.

O F
A S T R O N O M Y.

C H A P. I.

HAVING hitherto treated of *Arithmetick* in the four kinds of it, *Common*, *Decimal*, *Logarithmetical*, and *Instrumental*, and also of *Geometry*, both general in the Resolution of Geometrical Problems and Propositions; and in the producing, reducing, and measuring the three principal-Geometrical Figures: And likewise of *Geometry* in special applied to *Surveying* and *Navigation*: I come now in the third place to treat of the third *Mathematical Liberal Science*, which is *Astronomy*. And as *Geometry* may be said to measure the Earth, or this whole Terrestrial Globe of Sea and Land, by taking Heights and Distances

stances at Land in *Surveying*, and by taking Heights and Distances at Sea in *Navigati-on*: So *Astronomy* may be said to measure the Heavens by taking the Heights and Distances of those Heavenly Bodies, *viz.* The Planets and Fixed Stars.

But that these Heights and Distances here principally meant and intended, may be entirely handled, and that other things which concern *Astronomy*, may be the better understood, I shall premise these several particulars, and proceed in this Method.

1. I shall give you the number and names of these Heavenly Bodies, *viz.* the Planets and Fixed Stars.

2. I shall shew you their Order, both in the Ptolomaick and Copernican Systems of the World, and give you the Reasons given on both sides, *pro & con* for each Systeme.

3. I shall shew you their Periodical Revolutions.

4. I shall shew you the Magnitudes.

5. Their Heights or Distances from the Earth :

Which will lead me by the hand as it were in the last place to speak of these other Heights and Distances which I make the Sum and Substance of *Astronomy* as it stands in Analogy with *Geometry* in this place: And these, to wit, the latter sorts of Heights
and

and Distances are properly Questions of the Sphere, therefore I shall explain to you the *Projection of the Sphere in plano*, and the *Resolution of Spherical Triangles* grounded thereupon ; by which *Spherical Triangles* these *Astronomical Questions* are resolv'd ; as *Surveying* and *Navigation* are by *Plain Triangles*. But to proceed according to my proposed Method.

C H A P. II.

To give you the Number and Names of the Planets and Fixed Stars.

1. **A**S for the Planets, all agree as to the Number that there are 7, but which these seven are there is some difference ; *Aristotle*, *Ptolomy*, and the rest of the *Peripateticks* do number them thus, *Saturn*, *Jupiter*, *Mars*, *Sol*, *Venus*, *Mercury*, *Luna*, making the Earth the Center of the World : But *Pythagoras*, *Copernicus*, and the Ancient *Pythagoreans* do make the Sun the Center of the World, and this Earth to be a Planet instead of the Sun, and to move in the same Orb which is ascribed to the Sun by *Ptolomy*, and so numbers the Planets thus, *Saturn*, *Jupiter*, *Mars*, *Tellus*, *Luna*, *Venus*, *Mercury*, and *Sol* the Center, and

and some will not have the Moon to be a primary Planet, but only a secondary, and that all the rest of the Planets as well as this Earth have their Moons to move about them, and for *Jupiter* there are 4 observed to move about him, which they call *Satellites*; and *Saturn* also is observed to have his concomitants.

Secondly, For the number of the Fixed Stars, they are vulgarly and commonly conceived to be innumerable, yet *Mathematicians* do say they may be numbred, and do take notice only of 1025, or at most of 1161, comprized in 48 Constellations,

21. In the Northern Hemisphere.

15 In the Southern.

12 In the Zodiack, or the Zodi-
cal Constellations, being no

In all 48 other than the 12 Signs.

The

The Names of the Northern Constellations are

Containing Stars		Containing Stars	
1 Ursa Minor	7	13 Ophiucus, or Serpentarius	24
That in the end of the Tail is called the Pole Star.	14	14 Serpens	18
2 Ursa Major	27	15 Sagitta, or the Arrow	5
3 Draco	31	16 Aquila, or Vultur volans	9
4 Cephus	11	17 Delphinus the Dolphin	13
5 Bo. tes	22	18 Equiculus, or little Horse	4
6 Coronæ Boreæ	8	19 Pegasus, or the winged Horse	20
7 Eriogonassus or Hercules	29	20 Andromeda	23
8 Lyra	10	21 Triangulum the Triangle	4
9 Olor, or Cignus	17		
10 Cassiopeia	13	9 Constellations	Stars 117
11 Perseus	26		215
12 Aurigæ	14	21 Northern	332
12 Constellations	215 Stars.		

The

The Names of the 15 Southern Constellations.

N ^o .		N ^o .	
1	Cetus the Whale	22	9 Crater the Cup
2	Fluvius Eridanus	34	10 Corvus the Crow
3	Lepus the Hare	12	11 Centaurus
4	Orion	38	12 Lupus the Wolf
5	Canis Major	18	13 Ara the Altar
6	Canis Minor	2	14 Corona Austrina
7	Argo Navis	41	the South Garland.
8	Hydra	25	15 Piscis Austrinus
8	Constellations.	7	Constellations.
Stars	192	Stars	102
		8	192
		15	South Constellat.
			294

The Names of the 12 Zodiatical Constellations.

N ^o .		N ^o .	
1	Aries the Ram	13	7 Libra
2	Taurus	23	8 Scorpio
3	Gemini	18	19 Sagittarius
4	Cancer	9	10 Capricornus
5	Léo	27	11 Aquarius
6	Virgo	26	12 Piscis
6	Constellations	6	Constellations.
Stars	116	Stars.	154
		6	116
		12	Constellations.
			270

These

These are the 48 Constellations taken out of Mr. *Moxons* Book *De Globis*, where you may find their several Poetical Stories, which are not unpleasant to read: And thus much shall serve for the Number and Names both of the Planets and Fixed Stars.

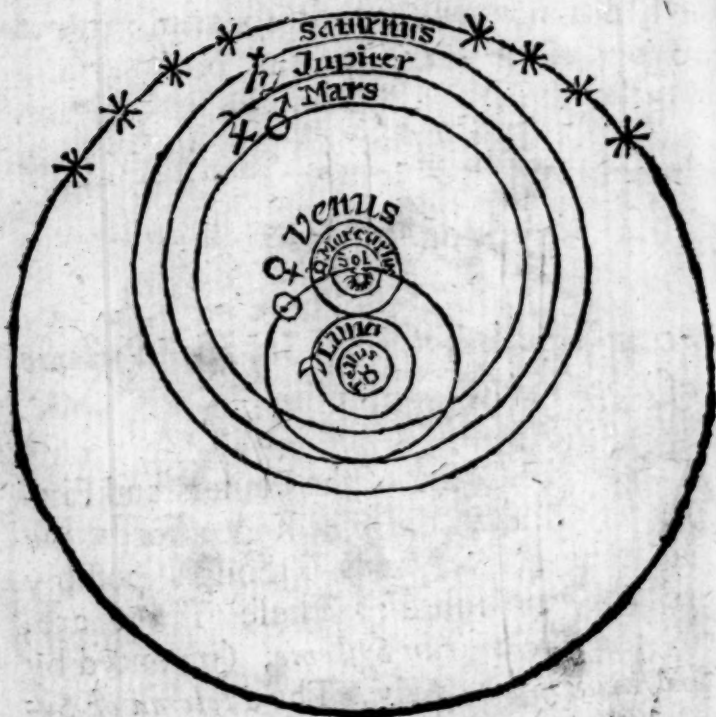
C H A P. III.

Concerning their Order in the Systeme of the World.

AS to the Order of the Planets and Fixed Stars, my Reading affords me three several Systemes (though possibly there may be more :) These Three are, First, the *Tychonican Systeme*, (invented by *Ticho Brahe*.) Secondly, The *Ptolomaick Systeme* (invented and maintained by *Ptolomy* and the *Peripateticks*.) Thirdly, The *Copernican Systeme*, being the Antient Opinion of *Pythagoras* and his Followers, but revived by *Copernicus*, and since imbraced by most Modern Mathematicians, though Vulgar Capacities do take it for a strange thing, *monstrum Horrendum*, &c. I shall give you these three several Systems in order, and first, the *Tychonican*.

This

The Tyconican Systeme.

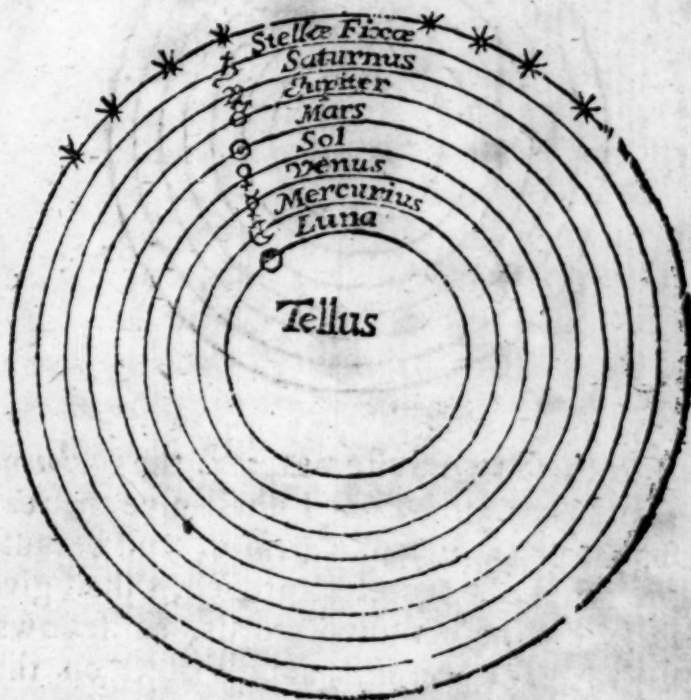


This Systeme was Invented by the Illustrious *Ticho Brahe* a Nobleman of *Denmark*, Lord of *Knudthorp* in the Island of *Shonen*, not far from *Elsenbourgh*, who framed this Hypothesis as a mean betwixt *Ptolemy* and *Copernicus*.

By this he supposeth that *Mercury*, *Venus*, and all the other Planets (except the Moon) in their motion, respect the Sun as their Center: So that *Saturn* in opposition to the Sun

Sun is nearer the Earth than *Venus* in Apogæon, and that *Mars* in opposition to the Sun, is nearer to the Earth than the Sun it self, as may appear by Inspection from the Hypothesis it self.

The Ptolomaick Systeme.



The Copernican Systeme.



For these two Systemes, viz. the *Ptolomaick* and the *Copernican*, I shall give the reasons *pro & con* out of *Varenius*, and because they cannot be better expressed, I shall give you them in his own words, as, follows, *Geographia Generalis* abbreviated on this manner, pag. 34, 35, 36, 37, 39, and 40.

Rationes Copernicani Systematis.

1. Quoniam tantus est numerus stellarum quæ horis 24. circa Tellurem circuitum videntur perficere, & hæc apparentia per
unius

unius Telluris (manentis in suo loco) motum explicari potest, ideo magis rationi consentaneum est hunc statuere quam illum; sicut nobis in Navi sedentibus, & appropinquantibus ad stationem multarum Navium quæ apparent nobis appropinquare, non ideo hisce motum ascribimus; & omnino cum natura non soleat facere per plura quod per pauca potest, virisimile est, in hac quoq; re illud observatum esse.

2. Quia incredibilis & omnem cogitationem superans celeritas motus stellarum illius foret: etenim cum infinito fere spatio a Tellure absint, & vastissimus orbis illis percurrendus sit, uno horæ minuto ad minimum per Centena millia milliarium ferri deberent. Contra, si Telluri motus ascribatur hic primus, manet hæc in suo loco, neq; de minima celeritate timendum est, quia Tellus circa suum axem rotatur.

3. Accedit argumento huic majus Robur si comparemus vastitatem corporum celestium cum terrestri. Etenim cum Sol ad minimum ducentes major sit Tellure, stelle vero fixæ vel millies majores, cui non verisimilius fiat Tellurem rotari circa suum Axem motu naturali, quam tanta corpora incredibili pernecitate de loco in locum moveri?

4. Quoniam omnes Astronomi illustriores cum Tychone coacti apparentis jam

negant Orbes solidos; quibus antiqui ad faciliorem stellarum motus Hypothesin utebantur. Ideo multo incredibilior videtur illarum circumlatio circa Tellurem. Negant autem Orbes solidos, quia si hi essent, concedenda esset penetratio orbium mutua, cum quidam planetæ in alterius alicujus Sphærâ deprehendantur frequenter.

5. Nulla Ratio reddi potest cur Stellæ circa Tellurem moveantur, cum contra cur Terra & reliqui planetæ circa solem cum reliquis planetis moveantur, aliqua possit dari Ratio.

6. Quia nec Polus, nec Axis est realis circa quem Stellæ ferri ponuntur; contra in Tellure & Polus & Axis est.

7. Quia multo facilior est Navigatio ab occidente in orientem, quam ab oriente in occidentem. Etenim ex Europa in Indiam navigatur mensibus circiter quatuor, ex India in Europam reditur sex mensium spatio circiter. Nimirum quia in illa Navigatione in eandem plagam cum Tellure moventur, in hac vero in contrariam.

8. Quia omnes apparentiæ cœlestes, ortus, occasus syderum, diierum incrementum, &c. possunt explicari, si Tellurem moveri ponamus.

Imprimis autem hujus Hypothesi commoditas & necessitas conspicitur in admirandis illis Planetarum affectionibus ad quas explicandas

candas *Ptolemaici* multos circulos *Epicyclos* & eccentricos sine ulla ratione excogitare coguntur : *Copernicani* autem illas ex Telluris motu secundo circa solem facili negotio deducunt ita, ut manifestam illarum causam reddant, & adeo facilem ut vel indocti illam capere possint, Nimirum.

1. Cur Planetæ interdum retrogradi videantur, & quidem *Saturnus* sæpius & diutius quam *Jupiter*, *Jupiter* quam *Mars*, &c. interdum celeriori motu ferri, interdum stationarii esse.

2. Cur *Venus* & *Mercurius* nunquam possint tota nocte conspiciui esse

3. Cur *Venus* nunquam majori a sole intervallo, quam sexaginta graduum, *Mercurius* non majori, quam triginta graduum intervallo discedat, & ideo nunquam oppositi conspiciantur soli.

4. Cur *Venus* ejusdem diei & vespere post solem & mane ante solem possit conspici.

Plures apparentias afferre juperfedeo : sed illæ præcipue sunt quæ per hunc motum Telluris adeo apposite & jucunde explicentur, ut admirandum potius esset si Tellus non moveretur talibus apparentibus Phænomenis.

9. Sol non tantum fons lucis est, quæ tanquam clarissima fax illuminat Tellurem, Lunam, Venerem & reliquos sine dubio planetas, sed etiam focus caloris & vitalis

spiritus quo totum hoc universum foveri & sustentari videtur. Ideo medium locum omnium obtinere, & hos circa eum moveri probabile.

10. Sole collocato in medio redditur aliqua causa quare reliqui planetæ & Tellus circa eum ferantur, nimirum quia sol vastissimum corpus est, & magnis viribus præditum, ideo reliquos planetas ad motum excitat.

11. Solem circa Axem suum rotari probant observationes Galilæi & Scheineri de maculis solaribus. Hac igitur ratione reliquis planetis circumeundi causa existit, nec videtur ei alius motus attribuendus.

12. Si Telluri locum inter Martem & venerem, soli vero centrum attribuamus, apte responderet motus singulorum planetarum distantia a Centro, quod in Ptolemaica Hypothesi non fieri patet ex motuum solis, Veneris & Mercurii consideratione.

Verum enim vero *Aristotelici* cum *Ptolemaicis* sententiam *Pythagoricorum* pluribus impugnat Argumentis & suam Hypothesin probare conantur hisce rationibus.

Rationes Ptolemaici Systematis sequuntur.

1. Terram esse ineptam ad motum propter gravitatem; & Gravia feruntur ad Centrum mundi, Tellus autem gravissimum corpus

corpus est, itaq; centrum illud occupat.

2. Gravia a Tellure discederent versus centrum universi, nisi in Tellure hoc centrum esset.

3. Partes Telluris moveri naturaliter motu recto ad centrum, ergo circularem motum esse contra naturam ejus.

4. Si Tellus moveretur, lapidem e turri demissum non posse cadere ad pedem turris, vel globum e tormento explosum versus orientem ad metam aliquam (vel etiam si avis quædam ad orientem volaret) non posse hanc attingere, si meta cum tota tellure versus orientem moveretur, vel saltem celeriores fore attactum, si ad occidentem emissus esset globus.

5. Neq; turres neq; ædificia consistere posse, sed propter illum Telluris motum collapsura esse, neq; homines a vertigine immunes fore.

6. Quia videmus (inquiunt) stellas mutare locum, non autem Tellurem.

7. Quia Tellus est in centro mundi: centrum autem non movetur.

8. Quia sacræ literæ stabilitatem Telluris confirmant.

Ad hac Argumenta Copernicani respondere solent hunc in modum.

1. Ad primum negando totam Tellurem
P 4 - gravem

gravem esse; namq; gravitas est partium ad totum Homogeneum tendentia: Et talis gravitas quoq; in lunæ partibus & solis.

2. Et aliorum gravium motus non ad Centrum universi sed ad Corpus Homogeneum est, ut ex partibus Lunæ, Solis, magnetis probatur.

3. Motum illum rectum esse partium Telluris, non totius Telluris, atq; hujus circularem motum non impedire illarum rectilineam Lationem, quod declaratur a partibus Lunæ & Solis.

4. Ad hoc Argumentum responderetur triplici modo; 1. Primo enim gravia talia non ad Centrum primario feruntur, sed ad ipsam tellurem, & ideo brevissima linea ad superficiem ejus, brevissima autem est hæc quæ turri respondet: Sicut ferrum non ad centrum magnetis, sed ad magnetem tendit. 2. Totus aer adhæret telluri & cum hac movetur, ideo etiam talia gravia demissa simul hunc circularem motum acquirunt & moventur tanquam in vase. 3. Gassendus crebrâ experientiâ demonstravit, quod si ex moto corpore aliquid projiciatur, hoc Projectum etiam illo motu corporis moveri, exempli gratiâ lapidem dejectum è fastigio mali navis celerrime motæ, tamen non relinquenda nave, sed ad pedem mali decidere: Et è pede mali explosum perpendiculariter è iclopeto globum, rursus perpendiculariter decidere

decidere. Itaque allata objectio nihil valet :
*Or if a Man jump in a Ship, he will not be
able to pass further, whether he jumps with, or
against the motion of the Ship.*

5. Ad quintum Argumentum dicimus tale quid locum non habere : quia motus est æquabilis, nec in aliud corpus impinget, & ædificia tanquam corpora gravia & Telluri homogenea moventur tanquam in navi. Etenim in navi celerrimè etiam vel tardissime mota, si modo æquabilis sit navigatio hoc est sine fluctibus, & aquà planà, deprehendimus erecta corpora non everti, imo pocula vino plena nihil effundere.

6. Ad sextum, dicimus mutationem loci stellarum non sentiri, sed situs respectu nostri mutationem deprehendimus : potest autem hæc situs mutatio animadverti & esse, siue nos cum Tellure siue nobis immotis stellæ moveantur, vel etiam & nos & stellæ.

7. In septima objectione & major & minor falsa est, vel saltem dubia.

8. Ad Octavum respondetur. 1. Sacram scripturam in rebus Physicis Loqui secundum apparentiam & vulgi captum, exempli gratia quando Luna cum Sole dicitur magnum luminare quod ad noctem illuminandum creatum sit, cum tamen nec Luna magna sit respectu stellarum & Telluris, nec proprium lumen habeat, neq; omnibus no-

ctibus illuminat terram. Ita solem ab extremitate ire, & ad extremitatem redire dicit scriptura, cum tamen revera talis extremitas nulla sit. Ita in *Jobi* libro attribuitur Telluri figura plana & quadrata, cui columnæ sint suppositæ, quibus innitatur, quod quidem haudquaquam ita intelligendum esse, vel vulgus novit. Plura loca adduci possint : sed sufficiunt hæc ; namq; sacræ literæ nobis non ad philosophandum sed ad pietatem colendam concessæ sunt. Secundo Loca quædam scripturæ adduci solent, quæ non de immobilitate ejus, sed de constantia & duratione loquuntur, ut locus ille quem e *Jobo* attulimus.

Thus you have the Reasons both *pro & con* laid down by *Varenius*, both for the *Ptolemaick* and *Copernican Systemes* in reference to the order wherein these Planets and Fixed Stars are placed : I shall leave it to every one to be of what Opinion he sees the most reason for : But however cannot pass by what I find in that great Mathematician Bishop *Wilkins* after that he hath argued seriously for the *Copernican* ; he adds jocantly, That they that would have the Earth stand still, and the Sun to move, are like the Cook who would not roast his Meat by turning the Meat about to the Fire, but rather by turning the Fire round about the Meat. (*World in the Moon.*)

If the Reader desire to know more concerning these matters, let him read the foresaid Author, and *Vincent Wing* his *Harmonicon Cœleste*, and such as have treated largely of these matters. These may suffice for my present purpose, and so shall pass on to the Third particular propounded to be considered, which is the Motion or Periodical Revolution of these Heavenly Bodies.

C H A P. IV.

Concerning the Motions and Periodical Revolutions of the Planets and Fixed Stars.

According to the two different Hypothesis of *Ptolomy* and *Copernicus*, there are you may see two different Centers to their several Systemes, viz. the Earth and the Sun, the one standing still, the other is supposed to move in its proper Orb betwixt *Venus* and *Mars*: If then according to *Ptolomy* the Earth be the Center, then must the Sun move, and also the Fixed Stars, or the Starry Firmament.

1. The Sun must move about the Earth in 24 hours, making Day and Night, which according

according to its proper Orbs Distance from the Earth of 1142 Semidiameters must be above 7570 *English* Miles in a Minute of time, and finisheth his periodical Revolution thorough each side in the Zodiack, call'd his second motion in a year, or 365 days 6 hours, *ferè*, *Wing* 365 days 5 hours 48 Minutes, 55 Seconds, by this making Winter and Summer, Spring and Autumn.

2. The Earth supposed the Center, the Starry Firmament must also move round about the Earth in 24 hours, and the motion of the Starry Firmament being proportionable to its distance from the Earth, a Star in the Equator must move 12598666 *English* Miles in an hour, or 209974 in a Minute, so that if an Horseman should ride every day 40 Miles, he could not ride such a Compass in a 1000 years as the Starry Firmament moves in an hour, which is more than if one should move about the Earth 1000 times in an hour, and quicker than possible can be imagin'd; and if a Star should fly in the Air about the Earth with such a prodigious quickness, it would burn and consume all the World here below: This made *Copernicus* not unadvisedly to attribute this motion to the Earth, and that the Orb of the Fixed Stars stands still, as well as the Stars in the said Orb are fixed in their places and due distances.

from

from one another. Supposing then with *Copernicus* the Sun to be the Center of the Universe; (but allowing it withal a circumrotation upon its own Axis) I say, supposing the Sun the Center, there must be ascribed to the Earth at least a two-fold motion: The first is its Diurnal motion, answerable to the *Ptolomaick* Diurnal motion of the Sun and Stars: This performed upon its own Axis in 24 hours, and constitutes day and night. The second motion of the Earth is his motion *a loco in locum* in its proper Orb betwixt *Venus* and *Mars*, as you may see in the Systeme, and this is performed in a year, or 365 days, 6 hours, *fere*; or 36 d. 5 h. 48'. 55". instead of the periodical revolution of the Sun according to *Ptolomy*, and this constitutes the 4 seasons of the Year, Spring, Summer, Autumn, and Winter; as is excellently demonstrated in Bishop *Wilkins's* *World in the Moon*.

Now as to the former of these motions, or *secundum primum Telluris motum* upon its own Axis; you must know that every place upon the Terrestrial Globe (as you may plainly see by turning about an Artificial one) doth not move alike, but places under the Equinoctial do move the fastest or quickest, and nearer either of the Poles they move slower, in such proportion as the

the

the Degrees in each parallel of Latitude are greater or less : Therefore to know how many Miles in an hour any particular place is carried by this motion, take this Proportion or Analogy. As the Radius is to the Co-sine of the Latitude ; so is 15 *d.* or 900 *English* Miles to the Number of Degrees or Miles required.

As for Example, *Amsterdam* is in the Latitude of 52 *d.* 23 *m.* its complement, or co sine is 37 *d.* 37 *m.* so that as 90, or Radius is to 37 *d.* 37 *m.* so is 900 to 548 *English* Miles ; or so is 225 *German* Miles, (4 *English* miles making one *German*) to 137 *German* Miles ; or so is 15 degrees to about 9 degrees, which *Amsterdam* moves in an hour, or 9 of these *German* miles in 4 minutes of time : Now how these Numbers 15, 225, and 900, come to be the third number in this proportion ; note that 15 *d.* is an hour in the *Æquator*, these multiplied by 15, produceth 225 *German* Miles, and multiplied by 60, produceth 900 *English* Miles, the *Æquator* passing so many Miles or Degrees in an hour, and so to run the whole circumference in 24 hours, 21600.

Thus much shall suffice to have spoken of the motion of the Earth, or the Sun, and of the Fixed Stars ; for the rest,

Luna is the lowest of all the Planets according to *Ptolomy*, or only a secondary Planet according to some *Copernicans*, finishes her course in 27 days, and almost 8 hours.

Mercury finishes his Revolution about the Sun in 87 days, 23 hours, 15 Minutes, 53 seconds, according to *Kepler*, and is never distant from the Sun above 30 degrees.

Venus performs its course in 224 days, 16 hours, 49 minutes, 24 seconds, and is never distant from the Sun above 60 degrees.

Mars finishes his course in 2 years, or more strictly in 686 days, 23 hours, 27 Minutes, and 30 seconds; or according to *Wing* in 1 year, 321 days, 23 hours, 32 minutes.

Jupiter moveth through the Zodiack in the space of 11 years, 10 months, and almost 16 days, or according to *Kepler* in 4332 days, 12 hours, 20 minutes, 25 seconds, according to *Wing* 11 years, 317 days, 14 hours, 49 minutes: His Satellites or Attendants circumvolveth him in time corresponding to their distances from him, the first and next him in 1 day, 18 hours; the second in 3 days, 13 hours; the third in 7 days, 4 hours, the fourth and outermost in 16 days, 5 hours.

Saturn, the highest of all the Planets finisheth his periodical course in 29 years, 9 months,

months, 15 days, according to *Alfraganus*, but more exactly according to *Kepler*, in 10759 days, 6 hours, 36 minutes, and 26 seconds, or according to *Wing*, pag. 63. *Harm.* in 29 years, 174 days, 4 hours, 58 minutes. Saturns Moon, or Concomitant is observed to move about him in 16 days, and note that the circumrotation of the Sun upon his own Axis is in 26 days, and the Earth in 24 hours, (together with the rest of the Planets that move about the Sun.) These all move from *West* to *East*, also *Saturn* and *Jupiters* concomitants about them do keep the same course from *West* to *East*.

And besides these, it is not improbable but that every primary planet hath his proper Revolution upon his own Axis (as the Sun and Earth have theirs :) and that they have their Moons or Concomitants moving about them as our Earth hath: Nay, my Author adds that it is not unlikely but that the Sun being of the same matter with the Fixed Stars, they also may all have their Planets or Worlds moved about him, as our Sun hath his. And *Des Cartes* hath shown us (saith Doctor *Power*,) That every Fixed Star is a Sun, and is set in the Center of a *Vortex* or Planetary Systeme as ours is; and that they are as far remote one off another, as ours is off them; and that all our whole Planetary *Vortex* shrinks into nothing

thing, if compared with these innumerable Systems above us.

No question but all these Planets are illuminated by the Sun (as is observed by the Telescope :) And that the Earth shines at a Distance with like splendor as her fellow Planets, as will easily appear by her illuminating the Darker part of the Moons subvolvane or lower Hemisphere, as is commonly seen a little before or after the change, for then to the Moon the appearance of the Earths light is near the full : But to make no further Digression. I come in the next place to shew the several magnitudes of these Cœlestial Bodies.

C H A P. V.

Concerning the Magnitude of the Planets and Fixed Stars, and their Distances from the Earth.

IT is not my Intention to shew you the Theory or Way of Calculation, or finding these Magnitudes, and Distances, or the several motions of the Planets : These require a large Treatise of it self, and is learnedly and largely done by Mr. *Vincent Wing* in that excellent Book of his *Harmonicon Cœleste*,

Caeleſte, to which I refer the Reader: And as I have done their Motions, ſo ſhall only *per Tranſitum* lay down their Magnitudes and Diſtances as I find them in Authors, and ſo proceed to queſtions of the Sphere, which is principally here intended. For their Magnitudes, take theſe Obſervations.

I. Theſe Planets have their ſeveral Orbs wherein they move, the Semidiameter of which Orbs compared to the Semidiameter of the Earth, have this proportion.

		Deg. Min.	
Of what Parts the Semidiameter of the Earth is one, of the ſame the Semidiameter of the Orb of	Luna	48	56
	Mercury	116	3
	Venus	641	45
	Sol	1165	23
	Mars	5032	4
	Jupiter	11611	31
	Saturn	17225	16

2. The Diameter of	Saturn	Compared with the Diameter of the Earth, is as	9	to	2
	Jupiter		32	to	7
	Mars		7	to	6
	Sol		11	to	2
	Venus		3	to	10
	Mercury		1	to	28
	Luna		5	to	17

The Diameter of the Sun compared with the Diameter of the Moon beareth the ſame proportion that is betwixt 187 and 10.

The

The Diameter of a Star of the	1	Magnitude is to the Earths Dia- meter, as	119 to 4
	2		269 to 60
	3		25 to 6
	4		19 to 5
	5		19 to 36
	6		21 to 8

3. The Magnitude or Proportion of the Body or Globe of these Planets to the Globe or Body of the Earth according to the best observations of Mr Vincent Wing in his *Harmonicon Cœleste*, lib. 3. chap. 25, 26. fol. 95, 96. are as follows.

Saturn } is greater than { $9\frac{2}{3}\frac{8}{17}\frac{2}{6}$ times.
Jupiter } the Earth { $9\frac{2}{9}\frac{2}{8}\frac{2}{9}$ times.
Sol } { 343 times.
and greater than the Moon $15765\frac{1}{3}\frac{0}{0}$ times.

Mars } is lesser than { $55\frac{6}{3}\frac{6}{2}\frac{1}{1}$ times.
Venus } the Earth { $26\frac{6}{1}\frac{0}{2}\frac{7}{6}\frac{7}{7}$ times.
Mercury } { $117\frac{2}{2}\frac{7}{7}\frac{1}{4}\frac{1}{4}$ times.
Luna } { $45\frac{2}{3}\frac{2}{0}$ times.

There are divers other Authors which do much differ from him in these Magnitudes, but I take him to be nearest the Truth.

The Magnitudes of the Fixed Stars, or their Proportions compared with the Globe of the Earth, which take as follows out of *Moxon de Globis*:

A Star of the

1
2
3
4
5
6

Magnitude is to
the Globe of
the Earth.

As 6859 to 64
As 19465109 to 216000
As 15625 to 216
As 6850 to 125
As 1685159 to 46656
As 9261 to 512

Therefore big-
ger than the
Earth.

Times.
107 $\frac{1}{2}$
90 $\frac{1}{2}$
72 $\frac{1}{2}$
54 $\frac{1}{2}$
36 $\frac{1}{2}$
18 $\frac{1}{2}$

This

This may seem incredible to visual sense, but if the vast Distance of those Huge Bodies, and the Diminutive quality of Distance be considered, reason will be rectified : For according to Mr. *John Dee's* computation, the Starry Firmament is distant from the Earth $20081\frac{1}{2}$ Semidiameters, which according to 60 Miles in a degree is 69006540 Miles : As is noted in its proper place.

And well may the Fixed Stars seem little to us ; for Doctor *Power* says, That if one stood in the Firmament to behold this Earth of ours it could never be seen at all, and therefore if it were annihilated, would never be missed : So much doth Distance diminish the Object.

Now for their Distances from this Globe of the Earth, I shall give you them from *Varenius* in his own words, *Geog. gen. lib. 1. c. 6.*

Si queras quantum Tellus & nos in Tellure existentes, distemus a Planetis, sciendum est, Non esse unam & eandem perpetuo distantiam sed singulis diebus mutari, & propterea Astronomi tres distantiarum gradus recensent minimam, mediam, maximam. Media Distantia Telluris a reliquis Planetis juxta plurimos Astronimos est hac.

<i>A Luna distat Semidiametris suis</i>	60
<i>A Mercurio</i>	110
<i>A Venere</i>	700
<i>A Sole</i>	1150
<i>A Marte</i>	5000 circiter
<i>A Jove</i>	11000 circiter
<i>A Saturno</i>	18000

Verum enim vero omnino incerta est Martis, Jovis, & Saturni & Stellarum fixarum distantia, ab defectum parallaxis. In Copernicanâ Hypothesi distantia variatur non tantum a Planetarum motu sed etiam a motu, ipsius Telluris.

John Seller's Atlas Cœlestis.

1. The *Sun* performs his Revolution upon his own Axis in 26 days.
2. *Mercury* performs his course in his Ellipsis in 88 days: His proper diurnal motion is 4 d. 5'. 12". the Circuit of his Sphere is 12059773 miles, so that he wheels in a day 137040 miles, in an hour 5710 Miles, and in a Minute 91 miles.
3. *Venus* makes her periodical Revolution in her Ellipsis about the Body of the Sun in 224 days. It is from the Sun to the Sphere of *Venus* 3636104 miles, hence the Circuit of her Sphere is 22855911 miles. Her mean Diurnal Motion is 1 d. 36'. 8". so that she moves in a day 101712 miles, in an hour 4238, in a minute 70.
4. The

4. The Earth accomplishes her Revolution in 365 days, 5 hours, and 59 minutes, which according to *Copernicus* Systeme is placed berwixt the Orbs of *Mars* and *Venus*. It is from the Sun to the Body of the Earth 5021896 Miles. The circuit of her Sphere is 31560207 miles, her Diurnal motion 39 minutes, 8 seconds. A Degree of a great Circle upon the Earths Surface is commonly reputed 60 miles, but by Mr *Normood's* experiment is found to be 69 miles.

5. *Mars* performs his Revolution about the Sun in one year, 321 days, 22 hours, and 20 minutes. It is from the Sun to the Body of *Mars* 7635292 miles. The Circuit of its Sphere is 47993264 miles, his Diurnal motion 31 minutes, 27 seconds, so that he wheels in a day 69842 miles, in an hour 2910 miles, in a minute $48\frac{1}{2}$ miles.

6. *Jupiter* runs his course in 11 *Egyptian* years, 615 days, 14 hours, and 30 minutes. It is from the Sun to *Jupiter* 26179152 Miles. The Circuit of his Sphere is 164554670 miles, and his Diurnal motion about the Sun is 4 minutes, 59 seconds. Hence he wheeleth every day 17996 miles, every hour 1583 miles, and every minute 26 miles.

7. *Saturn* performeth one Revolution about the Sun in 29 *Egyptian* years, 162 days,

days, 1 hour, 58 minutes. It is from the *Sun* to *Saturn* 47833576 miles, the Circuit of his Sphere is 300668192 miles, his proper daily motion in 2 minutes, 0 seconds. Therefore he wheeleth in a day 15959 miles, in an hour 1498, in a minute 25 miles. About his Body is a bright flat Ring which encompasseth him about.

The *Moon* is a secondary Planet, and retains the Earth for her Center, about which she performs her Revolution in 27 days, 7 hours, 43 minutes. It is from the Center of the Earth to the Moon 203236 Miles. The Circumference of her Sphere is 1277483 miles. Her Diurnal Motion is 13 degrees, 10 minutes, 35 seconds, so she wheeleth about in a day 46757 miles, in an hour 1948 miles, and in a minute 30 $\frac{1}{2}$ miles.

The Fixed Stars have a Motion or Circumrotation upon their own Axis, and besides a Motion of Revolution from West to East, which in a year is not less than 49 seconds, nor greater than 51 seconds, that it seems most probable that their Annual Motion is 50 seconds, 40 thirds, whence it follows that they compleat not one Degree in the Ecliptick sooner than in 71 years, and $\frac{1}{104}$ or 19 days, and 12 hours in a manner, but the whole Circle of 360 degrees they run not through in less than 25579

Sidereal

Sidereal years, which is *Annus Magnus Platonicus*, (though by the Antients computed to extend to 36000 years.)

But according to the Opinion and Observation of the Learned *Ticho Brahe*, as you may find in *Mathematical Recreations*, fol. 220. Their Distances are thus.

From the Center of the Earth	} to the	Moon	} is near	56 Semidiamet.
		Sun		or 192416 miles
		Starry Firmament		1142 Semidiam.
				or 3924912 mil.
				14000 Semidi.
				or 48184000 mil.

But according to Mr. *Dees* Computation 20081½ Semidiameters, or 69006540 Miles : from whence these Corollaries do follow.

I. Suppose that a Man should go 20 Miles in Ascending towards the Heavens every day, he should be above 15 years before he could attain to the Orb of the Moon.

2. If a Millstone should descend from the place of the Sun 1000 Miles every hour, (which is above 15 Miles in a Minute, far beyond the proportion of Motion) it would be above 163 days before it would fall down to the Earth.

Q

3. The

3. The Sun makes a greater way in one day than the Moon does in 20 days, because that the Orb of the Suns Circumference is at the least 20 times greater than the Orb of the Moon.

And thus much shall suffice to have spoken concerning the Number, the Names, the Order, the Motions, Magnitudes and Distances of these Heavenly Bodies, the *Planets* and *Fixed Stars*.

C H A P. VI.

Of the Sphere.

I Come now to speak of the *Sphere*, and of Questions of the *Sphere*, which I make the sum and substance of *Astronomy*, as it stands in Analogy with *Geometry* in this place: For as *Surveying* is taking Heights and Distances at Land, *Navigation* at Sea, and are performed by the Resolution of *Plain Triangles*: So this part of *Astronomy* is taking Heights and Distances in the Heavens (as is before hinted) and is performed by the Resolution of *Spherical Triangles*, or by *Projection* of the *Sphere* it self, which is the ground of these *Spherical Triangles*. And that I may proceed the more metho-

methodically, and be more clearly understood.

1. I shall first shew you what these Questions of the *Sphere* are, and how they may be termed taking Heights and Distances.

2. I shall shew you how these Questions may be resolved.

I. By Trigonometrical Calculation, or the Resolution of Spherical Triangles.

II. By Projection of the Sphere : 1. More Natural, by the Globe it self. Or, 2. More Artificial, by Projection of the Sphere in *plano*, and to shew you these Questions may not only be resolved by the Globe, but also by *Planispheres*, *Quadrants*, &c.

III. And lastly, I shall insist more largely concerning that principal and most useful question of the Sphere, to wit, Dialling, or of finding the Hour of the Day by the Sun, and the Hour of the Night by the Moon and Stars, and shall herewith conclude the whole work.

S E C T. I.

First then, The Astronomical Questions of the Sphere are these,

1. *For Heights.* To find

1. The Height or Elevation of the Pole, or Latitude of the place. Q 2 2. The

2. The Altitude or Height of the Sun or Stars above the Horizon at any time, which are called (particularly their Almucantaras, and their Meridian Altitude.)

2. For Distances.

3. The right Ascension of the Sun or Stars may be also called an Height, because it is the Meridian Altitude, or Height of the Equinoctial above the Horizon: But may be also called a Distance, because the Distance or Number of Degrees contained betwixt the first Degree of *Aries*, and the Degree of the Equinoctial that comes to the Meridian with the Sun or Stars is the Right Ascension of the Sun or Stars in Degrees and Minutes.

4. The Oblique Ascension of the Sun or Stars, though it sounds like an Height, is more properly a Distance, because it is the Distance in Degrees of the Equinoctial betwixt the first point of *Aries*, and that very point of the Equinoctial that rises or cuts the Horizon with the Sun or Stars.

5. The Ascensional Difference is in plain *English* no more but the Distance betwixt the Oblique and Right Ascension.

6. The Suns Meridian Distance North or South from the Equinoctial, or of a Star from the Ecliptick is called their Declination.

7. The

7. The Suns distance from any of the 12 signs is called his place in the Zodiack.

8. His Distance of rising or setting from the due point of East or West, is called his Amplitude.

9. The Suns Distance at any time from any point of the Horizon is called his *Azimuth*, or Angle of the *Suns position*.

10. The *Suns* distance from the next Equinoctial point *Aries* or *Libra*, requires no Explanation.

Lastly, His Distance from the Meridian shews the hour of the day, and the Moon and Stars distance from the Meridian shews the hour of the night. And all these in these respects may be properly called Heights and Distances in *Astronomy*.

S E C T. 2.

Therefore in the second place I shall shew you how to find these Heights and Distances.

1. By Trigonometrical Calculation, or the Resolution of Spherical Triangles.

2. By Projection of the Sphere, either Globular, or in *plano*, in whole, as in the Globe it self and Planispheres; or in part, as in Semicircles, Quadrants, and the like.

For the first of these, to shew you how these Questions may be resolved by the Resolution of Spherical Triangles, I shall endeavour to do these two things.

1. I shall speak something concerning Spherical Triangles, and the way how to make and measure them.

2. I shall give you the Trigonometrical proportions how to find these several Astronomical Heights and Distances before-mentioned by Sines and Tangents.

S E C T. 3.

Of Spherical Triangles.

As *Plain Triangles*, so *Spherical* consist of six parts, *viz.* three Angles, and three sides; the Angles of both are measured and protracted by a line of Chords. The difference betwixt them lies only in this, that the sides of Plain Triangles are Right lines drawn by the fiducial edge of a Ruler, and measured by a Line of equal parts: whereas the sides of Spherical Triangles are Arches of great Circles (that is Arches of any Circle to the same Radius) protracted upon the Center of the same Arches or Circles, and measured by a Line of Chords after the same manner, that all Circles and Arches are measured by Degrees and Minutes.

And

And as the Proportions betwixt the Sides and Angles of Plain Triangles are found by Numbers and sines : As for Example, the Artificial sine of any Angle is proportionable to the Logarithmical Number of its opposite side ; and on the contrary, the Logarithmical Number of any side is proportional to the Artificial sine of its opposite Angle : So are the sides of spherical Triangles (being Arches of great Circles ; and the Angles also measured at their Quadrantal Distance, or at the Distance of the Radius from the Angular point,) and the proportion between these Sides and Angles are found by Sines only by this Rule. *Harm. Cæl. fol. 17.*

In every, or in any spherical Triangle, the sines of the sides are proportional to the sines of their opposite Angles, and the sines of the Angles are proportional to the sines of their opposite sides.

And in these particulars lies the difference betwixt plain and spherical Triangles, and betwixt the protracting or making, and the measuring of the same : More particularly,

1. To measure or take the Quantity of any Angle of a Spherical Triangle, and to protract the same by the Projection.

Lay a Ruler to the Angular point, and
Q 4 the

the Extremity of the sides containig the Angle, they being continued to Quadrants, and note where the Ruler cuts the Meridian or outward Circle, at both which places make marks upon the Meridian: The Distance between those two marks measured upon your Line of Chords, shall give you the quantity of the Angle required.

And to Protract an Angle, do thus,

First, Taking 90 degrees or Radius from your Line of Chords, describe a Circle or part of a Circle, so much of it as you have occasion for, and making the Center to be the Angular point, prick out in the circumference so many Degrees and Minutes from the same Line of Chords as you would have your Angle to contain: Then at the distance of the Radius from these marks in the same Circumference placing one foot of your Compasses, with the other strike an Arch through the mark and the Center, do the like with the other mark: These Arches at the Center shall include an Angle of a Spherical Triangle, containing the same Number of Degrees as were markt out in the Circumference. The like Ratio holds in any other Angle. But,

2. *To Measure, or take the quantity of any side of a Spherical Triangle.*

And to protract the same, is more difficult, and requires in the first place (seeing that all Circles have their proper Poles) That the Poles of those Circles (whereof those sides are Arches) be found out.

3. *Now to find out these Poles.*

Note that the Pole of every great Circle is 90 degrees, or a Quadrant of a Circle and distant from the Circle it self upon that line which cutteth the Circle at Right Angles. Thus in a Projection of the Sphere the Poles of all the hour-Circles are upon the *Æ*-quinoctial, and the Poles of all the Azimuths upon the Horizon: The Poles of the Equinoctial are the Poles of the World; and the Poles of the Horizon, the Zenith and Nadir. Now to find the Pole of any Arch continued to a Quadrant, lay a Ruler by the extream ends of the said Arch, and mark where it cuts the Meridian or outward Circle: Then take 90 Degrees from your line of Chords, and set off that distance from the said mark; and lastly, lay a Ruler from the former end of the Arch to this latter mark, and where this crosses the Circle

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which

which cuts the Arch at right Angles, there is the Pole of the said Arch. To make it more plain by an Example.

Suppose you would find the Pole of the Hour-circle PDS in the Projection, lay a Ruler upon P and D, it will cut the Meridian or outward Circle in *e*. Then take 90 degrees of your Line of Chords, and set them from *e* to *f*. A Ruler laid from P to F will cut the Æquinoctial in *y*, so is *y* the Pole of the Circle PDS, and so of any other as the Pole of ZGN is ☉, &c.

4. *Thus having found the Pole of a Circle, or of the side of a Spherical Triangle, which is the Arch of a Circle, to measure the said side.*

Lay a Ruler upon the said Pole, and to the extreame ends of the side of a Triangle, note where the Ruler so laid cuts the Meridian or outward Circle at both ends of the side : That Distance taken in your Compasses, and measured upon the Line of Chords, will give you the Quantity of the side of the Triangle. As for Example in the Projection : Let it be required to find the side EZ of the Triangle ZEP, lay a Ruler to ☉, (the Pole of the Circle ZEN,) and the Angular point E, it will cut the Meridian in M, and a Ruler laid to Z will cut the

the Meridian in Z, so the distance MZ taken in the Compasses, and measured upon the Line of Chords, will be found to contain 78 degrees, and such is the quantity of the side ZE.

5. *Now to Protract a side of a Spherical Triangle, is taught by measuring of it.*

For first you are to find the Pole, then laying your Ruler from the Pole by the extremity of one end of the side, make a mark upon the Meridian or outward Circle, and make another mark at the extremity of the other end of the side; and lastly find out the Center of the Circle, to the same Radius; which will cut both the extremities of the side required, and it is done.

S E C T. 4.

Having thus shewn you how to protract and measure either Angle or side of a Spherical Triangle; and withal given you pag. 343 the Rule to find out the proportions; or by some of the 6 parts given, to find the rest, viz. That the sines of their sides are proportional to the sines of their opposite Angles, & *è contra*, which Rule is so general, that thereby you may resolve all, or most questions of the Sphere or Spherical Triangles.

Triangles: yet I think it not amiss to lay down the particular proportions by which these several Astronomical Questions of the Sphere are resolved and found by the lines of Artificial Sines and Tangents, or Logarithmetical Arithmetick,

How to find the Suns place,
And his Declination,
His Meridian Altitude, and thereby,
The Latitude.

All these you will find Taught in *Navigation*, pag. 254, 257, and 258, whereunto I refer the Reader; They being there handled as they are *Data*, or things given: And therefore some of these being given, I shall shew you here how they and other postulata's or quæsitâ's may be found in these following questions.

P R O B L. 1.

By the Suns shadow to find his Altitude.

Take a two-foot Rule, or any other staff divided into equal parts: And as the length of the shadow is to the length of the Rule or Staff (held perpendicular to the Horizon) upon the line of Numbers; so is the Tangent of 45, or Radius to the Suns Height required: As suppose the staff divided into 100 parts, the shadow is 83 of these

these parts, the extent betwixt 100 and 83 in the Line of Numbers will reach from Radius, or 45 upon the Line of Tangents to 50 d. 18 m. 26 seconds, the Suns Height above the Horizon.

P R O B L. 2.

To find the Suns Declination, vide pag. 258.

P R O B L. 3.

To find the Suns place you have also 257. But shall add this other Analogy, The Suns greatest Declination, and his present Declination given, to find his Distance from the next Equinoctial point, and thereby his place in the Ecliptick.

As his greatest Declination, (which is always 23 d. 30 m.) is to Radius or sine 90, so is his present Declination (which is also to be given) to his Distance from the next Equinoctial point: Suppose the Declination 11 d. 30 m. his distance is 30 d. from the next Equinoctial point, which is Aries, therefore his true place is just leaving Aries, and entring Taurus.

P R O B L.

P R O B L. 4.

The Suns greatest Declination, and Distance from the next Equinoctial point given, to find his Right Ascension.

As the Radius is to the Tangent of his Distance, &c. so is the Co-sine of his greatest Declination to the Tangent of his Right Ascension. Example, as 90 d. is to 66 d. 30 m., so is Tangent of 30 d. to the Tangent of 27 d. 50 m.

P R O B L. 5.

The Suns greatest Declination, and his present Declination given, to find his Right Ascension.

As the Tangent of his greatest Declination (23 d. 30 m.) is to Radius, so is the Tangent of his present Declination (which suppose 11 d. 30 m.) to the Sine of his Right Ascension 27 d. 50 m.

P R O B L. 6.

To find the Oblique Ascension.

If the Declination be North, subtract the Ascensional Difference from the right Ascension, and it will give the Oblique Ascension. If the Declination be South, add

add the same Ascensional Difference and Right Ascension together, and the Sum will be the Oblique Ascension.

P R O B L. 7.

To find the Ascensional Difference, the Latitude and Declination given.

As the Co-Tangent of the Latitude, (which suppose 38 d. 30 m. the Latitude being 51 d. 30 m.) is to the Tangent of the Suns Declination (20 degrees) so is the Radius (90 d.) to the sine of the Ascensional Difference (27 d. 14 m.) and so much the Sun riseth or setteth before or after six, according as the Declination is North or South, and this 27 d. 14 m. converted into time (by allowing 15 degrees to an hour, and one degree for 4 minutes of time) is 1 h. 49 m. and so much doth the Sun rise or set before or after the hour of six, according to the time or season of the year.

P R O B L. 8.

The Latitude of the Place, and Declination of the Sun given, to find his Amplitude.

As the Co-sine of the Latitude (38 d. 30 m.) is to Radius (90) so is the Sine of the Declination

clination (suppose 20 d.) to the sine of the Amplitude (33 degrees 20 minutes.)

P R O B L. 9.

To find the Suns Altitude, and first his Meridian Altitude, his Declination (only) given.

If the Declination be North, add the same to the Complement of the Latitude, and the Total of both shall be the Meridian Altitude. If the Declination be South, subtract the same from the Complement of the Latitude, and what remains shall be the Meridian Altitude.

P R O B L. 10.

The Latitude and Declination given, to find the Suns Altitude at 6 a Clock.

As the Radius (90 d.) is to the sine of the Suns Declination (20 d.) so is the sine of the Latitude (51 d. 30 m.) to the sine of the Suns Altitude at 6, viz. 15 d. 30 m.

P R O B L. 11.

The Latitude and Declination given, to find the Suns Altitude when he is upon the true East and West points.

As the sine of the Latitude (51 d. 30 m.) is to the sine of the Declination (20 d.) so is the Radius (90 d.) to the sine of the Suns

Suns Altitude, being due East or West
(25 d. 55 m.)

P R O B L. 12.

*To find the Suns Altitude at any time
assigned.*

First, Say as the Radius is to the Cotangent of the Elevation, so is the sine of the Suns distance from six to the Tangent of an Ark, which being subtracted out of the Suns distance from the Pole. I say again,

Secondly, As the Co-sine of the Ark found is to the Co-sine of the Residue of the Suns Distance from the Pole, so is the sine of the Elevation to the sine of the Height required.

P R O B L. 13.

*To find the Angle of the Suns Position, having
given the hour from Noon: The Latitude,
and the Suns Altitude.*

As the Co-sine of the Suns Altitude, (which suppose 78 d.) is to the sine of the hour from Noon (35 d. 36 m.) so is the Co-sine of the Latitude (38 d. 30 m.) to the sine of the Angle of the Suns Position at the time of the Question, (21 d. 45 m.)

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P R O B L. 14.

The Latitude of the place, and Declination of the Sun being given, to find the Angle of the Suns position at his Rising.

As the Co sine of the Declination (70 d.) is to Radius (90 d.) so is the sine of the Latitude (51 d. 30 m.) to the sine of the Angle of the Suns Position at the time of his rising.

P R O B L. 15.

Having the Suns Altitude, his Declination, and Distance from the Meridian (or the hour of the day) given, to find his Azimuth.

As the Co-sine of the Suns Altitude, (which suppose 64 d.) 4 m.) is to his distance from the Meridian (61 d.) or 4 hours, 4 m. before noon (viz. 7 hours 56 m.) so is the Co-sine of the Declination (78 d. 30 m.) to the Suns Azimuth from the South-Eastwards, (viz. 72 d. 22 m.)

P R O B L. 16.

The Latitude of the place and Declination of the Sun given, to find what Azimuth the Sun shall have at six a Clock.

As the Co-sine of the Latitude (38 d. 30 m.) is to Radius (90 d.) so is the Co-Tangent

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of

Tangent of the Declination (70 d.) to the
Tangents of the Suns Azimuth counted
from the North part of the Meridian (77 d.
14 m.)

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P R O B L. 17

*The Declination, Altitude, and Azimuth of
the Sun being given, to find the Hour of
the Day.*

As the Co-sine of the Declination (78 d.
30 m.) is to the Co-sine of the Altitude
(64 d. 4 m.) so is the Azimuth (72 d. 22 m.)
to 61 d. the hour of the day. This 61 d.
converted into time gives 4 hours, 4 mi-
nutes, which taken from 12 hours, the Re-
mainder is 7 hours, 56 minutes before
Noon: The time of the day required.

Mr. Leyburns Analogy is this,

As the Co-sine of the Declination (70 d.)
is to the sine of the Azimuth (146 or 34
degrees) so is the Co-sine of the Altitude
(78 d.) to the sine of the hour from
Noon 35 d. 36 m.

C H A P

C H A P. VII.

THUS you have the several Analogies & Proportions, whereby may be resolved these most useful Questions or Problems of the Sphere by the *Rule of Three*, wrought either by *Logarithmes*, pag. 31. or by the Lines of Artificial Numbers, Sines, and Tangents, pag. 41. taking this Observation along with you.

That (*As*) denotes, or is put before the first Number : (*Is to*) the Second : (*So is*) denotes the third : and (*To*) depotes or precedes the fourth, so that multiplying the second by the third, and dividing the Product by the first, produceth the fourth in the Direct *Rule of Three*, pag. 19. and multiplying the First by the Second, and dividing by the Third, produceth the Fourth by the Indirect or Reverse *Rule of Three*, pag. 19. where you may find both the said Rules.

Further you may observe, that the Suns Declination and Altitude is Assistant to find the Latitude : His Amplitude and Azimuth, to find the variation of the Compass : The Ascensional Difference to find the time of the Suns Rising and Setting : The right
Ascension

Ascension of the Sun and Stars is Assistant to find their coming upon the Meridian, and the hour of the Night. The Latitude, Declination, and the Suns Altitude, and Azimuth is Assistant and subservient to the finding the hour of the day: These two last put together, that is, the finding of the hour of the Day and Night is that which we call Dialling, concerning which, you will find something particularly in the Conclusion of this small Treatise.

Having now shown you how those questions of the Sphere may be resolved by *Trigonometrical Calculation*, and their several Analogies and Proportions also, by which they may be found by Arithmetical Operation. I come now to shew you how these may be done by several Projections.

First, by the Globe it self, being the most natural projection of the Sphere of all others whatsoever.

Secondly, By *Theakers Planisphere*, an Instrument of good use for this purpose, called a light to the Longitude.

Thirdly, By Mr. *Foster's* Quadrant, that is, a Quadrant with hour-Lines.

Fourthly, By the Projection of the Sphere in plano. And,

Lastly, I shall particularly insist more at large upon that Question concerning finding of the hour of the day by the Sun, and
of

of the hour of the Night by the Moon and Stars, which may be properly termed Dialecting, and with that conclude what I here intend in this Manual.

S E C T. I.

The Use of the Globe in questions of the Sphere.

Those who have a mind to be thoroughly instructed in the general Use of the Globes, both Cœlestial and Terrestrial, I recommend to them that most compleat Book written by Mr. Joseph Moxon concerning the Use of the Globes in the solution of Astronomical, Geographical, Nautical, Astrological, Gnomonical Problems; and in the Resolution of Spherical Triangles at large: What I here intend is after a very brief manner, to shew you how these Questions of the Sphere may be resolved by the Globe easily and speedily.

First then, you are to find the Latitude of the place where the Question is propounded.

Which to do by Observation is taught pag. 255. but if you desire to do it by the Globe, you are to bring the particular place to the Brazen Meridian, and the degrees reckoned thereupon from the Equator to the

the place whose Latitude you seek, is the Latitude whether North or South: And for the Longitude likewise you may find it by Observation, as in pag. 260, and by the Globe. Observe what Degrees of the Equator comes to the Meridian with the place, the same is the degrees of Longitude from the first Meridian.

In the next place you are to rectifie your Globe fit for use: Which to do,

1. When you rectifie the Globe to any particular Latitude, you must move the Brazen Meridian through the Notches of the Horizon, till the same Number of Degrees accounted on the Meridian from the Pole (towards the North point in the Horizon, if in North Latitude, and towards the South point, if in South Latitude) come just to the edge of the Horizon.

2. Next rectifie the Quadrant of Altitude, and screw the edge of the Nut that is even with the graduated edge of the thin plate to the same Number of Degrees as the Latitude is accounted on the Meridian from the Equinoctial; which will be the Zenith of the place.

3. Bring the degree of the Ecliptick the Sun is in that day to the Meridian, (which you shall learn to know hereafter) and then

then turn the Index of the Hour-Circle to the hour of 12 on the South-side the Hour-Circle, if in North Latitude & *contra*; and then is your Hour-Circle also rectified, fit to use for that day,

Lastly, To rectifie the Globe to correspond in all respects with the position of the Sphere, you must with a Needle placed in the bottom set the 4 quarters of the Horizon, *East, West, North, and South*, agreeable to the 4 Quarters of the World, and the Horizon by a Level you may set parallel to the Horizon of the World.

And positing the degree of the Ecliptick the Sun is in to the Height above, or depth below the Horizon the Sun hath in Heaven: Your Globe is made correspondent in all points with the frame of the Sphere, for that particular Time and Latitude.

The Globe thus rectified, you may find all the forementioned Problems on this manner.

1. Find the day of the Month in the wooden Horizon, and over against it you will find the degree of the Ecliptick, which the Sun is in that day: As suppose at *London* (where the Pole is elevated $51\frac{1}{2}$ degrees of Northern Latitude, the Globe rectified for the same Latitude, as also the Hour-Circle, and Quadrant of Altitude) over against

against the 10th of *May* you will see the Suns place in the Zodiack is 29 degrees of ζ *Taurus*,

2. Bring this degree which is the Suns place to the East-side of the Horizon, and its distance from the Equinoctial, or from the due East point is the Suns Amplitude, or so much as the Sun riseth before or after the Hour of Six 33 *d.* 20 *m.* from the East-point Northwards in this Example.

3. Look upon the Hour-Circle, and the Index will point to the Hour of Sun-rising, which in this Example is 12 Minutes after 4 in the Morning.

4. Observe what degree of the $\mathcal{A}$ equinoctial riseth with the Suns place, and that is his Oblique Ascension, reckoning from the Vernal Colure in the Cœlestial, and from the first Meridian in the Terrestrial Globe.

5. As you move the Globe round upon his Axis Westward, you may (at any time or hour which the Index of the Hour-Circle points to) know the Suns Altitude at that time, by bringing the Quadrant of Altitude to cut the Suns place: As at 53 *m.* past 8 in the Morning, you will find it just 40 degrees high above the Horizon: And,

6. The Quadrant of Altitude at the Horizon will shew his Angle of Position, or upon what point of the Compass it is.

7. The Suns Azimuth is the same with his Angle of Position.

8. And his Almicanter the same with his Altitude.

9. Bring the Suns place to the Meridian, and there the degrees intercepted betwixt the Suns place and the Horizon, reckoning upon the Brazen Meridian is the Suns Meridian Altitude : In this Example, (50 d. 30 m.)

10. The Distance betwixt the Suns place in the Ecliptick, and the Equinoctial is the Suns Declination reckoned also upon the Meridian : In this Example (20 d. 5 m.)

11. The Degree of the Equinoctial which comes to the Meridian with the Suns place, is his right Ascension, reckoning from the Vernal Colure, or first Meridian.

12. The number of degrees comprehended betwixt the Oblique and Right Ascension, is the Ascensional Difference reckoned upon the Equinoctial.

13. At any Degree of Altitude, either before Noon, or after Noon, the Index of the Hour-Circle points to the Hour of the Day.

14. And lastly, when the Suns place comes to the West-side of the Horizon, the Index of the Hour-Circle will point to the Hour of Sun-setting, and also will shew his Oblique Descension, and upon what Azimuth

Azimuth or Point of the Compass it sets, and how much before or after six.

What is here said of the Sun, the same is to be understood of any Star, using the said Star as you do the Suns place: As to Instance in these particulars: the Globe rectified as before.

1. By the Meridian Altitude of any Star, to find the Elevation of the Pole: Observe by a Quadrant or other Instrument the Meridian Altitude, and bringing the Star to the Meridian, place it upon the Globe to the Altitude observed, so shall the Number of Degrees intercepted between the Pole and the Horizon, be the elevation of the Pole.

2. To find the Right Ascension of any Star, bring the Star to the Meridian, and the Number of degrees betwixt the Vernal Colure, and the Meridian its Right Ascension; in like manner for his Oblique Ascension, or Oblique Descension.

3. To find the Declination of a Star, bring the Star to the Meridian and count the Number of Degrees betwixt the said Star and the Equinoctial, the same is the Stars Declination be it North or South.

4. The Azimuth and Almucantara, or which is all one, The Altitude and Angle of Position of any Star is found the same way as the Suns Azimuth and Almucantara,

as in 5, 6, 7, 8, fol. 369.

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5. To know any Star in Heaven, First, having a Theodolite, with a moveable Index, upon the Index fix a Two-foot Rule, or such a Thing Perpendicular; and at the Top of the Rule hang a Quadrant, then placing your Theodolite true North and South by help of a Needle, or a Meridian Line, you may by the Quadrant take the Almican-tara or Altitude; and the Index of the Theodolite will shew the Azimuth or Angle of Position: And observe by the Globe what Star there hath the same Azimuth and Almican-tara at the same time, and that is the same Star which you observed in Heaven: For if every Star on the Globe had a hole in the midst, and your Eye were placed in the Center of the Globe, you might by keeping your Eye in the Center, and looking through any Star on the Globe, see the same Star in the Heaven, which that on the Globe doth represent; for from the Center of the Globe there proceeds a strait Line through the Star on the Globe even to the same Star in Heaven.

6. The Meridian Altitude of a Star is found upon the Globe by bringing the said Star to the brazen Meridian, and reckoning how many degrees it is above the Horizon, the Globe rectified as before shown.

7. And by the Hour-Circle, turning about the Globe, you may observe at what Hour

Hour any Star will be upon the Meridian, to wit, when the same Star on the Globe touches the Brazen Meridian.

8. The Globe rectified, and the Altitude of a Star found by a Quadrant in the Heavens, and the same Star brought to the same degree of Altitude upon the Globe, by the Quadrant of Altitude, the Index of the hour-Circle will point to the hour of the night.

9. The Rising and Setting of the Stars are found as the Rising and Setting of the Sun: But this Rising and Setting admits of a threefold distinction:

1. *Cosmical*, when any Star riseth with the Sun it is said to rise Cosmically, and when any Star sets when the Sun riseth, it is said to set Cosmically.

2. *Acronical*, The Stars that rise when the Sun sets are said to rise Acronically, and the Stars that set with the Sun are said to set Acronically.

3. *Heliacal*, when a Star formerly in the Suns Beams gets out of the Suns Beams, it is said to rise Heliacally, and when a Star formerly out of the Suns Beams gets into the Suns Beams it is said to set Heliacally.

And what Stars do rise or set any of these ways, and when may be easily found by the Globe, as in *Moxons lib. 2. Probl. 35, 36, 37*, which I shall not here insist upon.

Lastly, I shall shew you how to find the Longitude and Latitude of a Star, and so conclude as to the Cœlestial Globe.

If the Star you enquire after be on the North-side the Ecliptick, you must elevate the North Pole $66 \frac{1}{2}$ degrees: If on the South-side you must elevate the South Pole as much, then bring the solstitial Colure to the Meridian on the North-side the Horizon, and screw the Quadrant of Altitude to the Zenith, which will be $23 \frac{1}{2}$ degrees from the Pole of the World, so shall the Ecliptick lye in the Horizon, and the Pole of the Ecliptick also lye under the Center of the Quadrant of Altitude: Now to find the Longitude of any Star, turn the Quadrant of Altitude about till the graduated edge of it lye on the Star, and the degree in the Ecliptick that the Quadrant touches is the Longitude of that Star: And the degree of the Quadrant of Altitude that touches the Star is the Latitude of the Star, so the Longitude of *Marchab*, a bright Star in the Wing of *Pegasus* will be found $\times$ 18 d. 56. and its Latitude 19 d. 26 m.

What hath hitherto been said concerns the Cœlestial Globe, I shall add a few propositions proper to the Terrestrial, which together will contain the principal Problems to be performed by the Globe.

1. *To find the Distance between any two Places on the Terrestrial Globe.*

The Graduated Edge of the Quadrant of Altitude applyed to both the Places, will shew the number of Degrees betwixt them, or with a Pair of Compasses pitch one foot in one of the Places, and the other foot in the other Place, and keeping your Compasses at that distance, apply the feet to the Equinoctial Line, and you will find the number of Degrees comprehended between them: As suppose the places were *London* and *Jamaica*, you will find $68\frac{1}{2}$ degrees, which multiplyed by 60 makes 4110 *English Miles*.

2. *To find upon what point of the Compass any Two Places are situate one from another.*

Find the two places on the Terrestrial Globe, and see what Rumb passes through them, or if no Rumb pass immediately through both, take that Rumb which runs most parallel to both the places.

3. *To know what a Clock it is in any other part of the Earth.*

Note that those places to the Eastward

of us have their day begin sooner than ours, and those to the Westward later: therefore first bring the place of your own habitation to the Meridian, and the Index of the Hour-Circle to 12, then bring the other place to the Meridian, and the Index will point to the Hour in the other place, be it sooner or later.

4. *Any place on the Terrestrial Globe given, to find its Antipodes.*

Bring the given place to the Meridian, so you may find its Longitude and Latitude, then turn about the Globe till 180 degrees of the Æquator pass through the Meridian, and keeping the Globe to this position, number on the Meridian 180 degrees from the Latitude of the given place, and the point just under that degree is the Antipodes; or bring the given place to the North or South point of the Horizon, and the point of the Globe denoted by the opposite point of the Horizon is the Antipodes of the given place.

5. *To find the Perecy of any given place.*

You must understand, the Perecy of any place are such as dwell about the same Hemisphere in the same Parallel from the Æquator, the one Eastern, the other Western, directly opposite to one another:

There-

Therefore bring your place to that side the Meridian which is in the South notch of the Horizon, and follow the parallel of that place on the Globe till you come to that side the Meridian which is in the Northern notch of the Horizon, and that is the Percecy of your place.

7. To find the Antecy of any given place.

The Antecy are such as dwell one against another, having the self same Meridian, and equal distance from the Æquator, the one in the Northern, the other in the Southern Hemisphere, therefore bring your place to the Meridian, and find its Latitude: Then if it have North Latitude, count the same number of Degrees on the Meridian from the Æquator Southwards: but if it have South Latitude, count the same number of Degrees from the Æquator Northwards, and the Point of the Globe directly under that number of Degrees is the Antecy of your place.

8. How to hang the Terrestrial Globe in such a Position, that by the Suns shining upon it, you may with great delight at once behold the Demonstration of many Principles in Astronomy and Geography.

Take the Terrestrial Ball out of the Horizon, and fasten a Thread on the Brazen

Meridian to the Degree of the Latitude of your place: By this thread hang the Globe where the Sun-beams may have a free access to it: Then direct the Poles of the Globe to their proper Poles in Heaven, and with a thread fastned to either Pole, brace the Globe so that he do not turn from his position: Then bring your Habitation to the Meridian, so shall your Terrestrial Globe be rectified to correspond in all respects with the Earth it self, so that with great delight you may behold.

1. How the counterfeit Earth (like the true one) will have one Hemisphere Sun-shine-light, and the other shadowed by the shining Hemisphere, you may see that it is day in all places that are scituate under it, for on them the Sun doth shine: And that it is Night at the same time in those places that are scituated in the shadowed Hemisphere; for on them the Sun doth not shine.

2. If in the middle of the enlightned Hemisphere you set a Spherick Gnomon perpendicularly, it will project no shadow, but shew that the Sun is just in the Zenith of that place, that is directly over the Heads of the Inhabitants.

3. If you draw a Meridian Line from one Pole to the other just through the middle of the enlightned Hemisphere, in all places under that Line, it is Noon; in those places

ces scituate to the West it is Morning, for with them the Sun is East; and in those Places scituate to the East it is Evening, for with them the Sun is West.

4. So many degrees as the Sun reaches beyond either the North or South Pole, so many Degrees is the Declination of the Sun either Northwards or Southwards, and in all those places comprehended in a Circle described at the termination of the Sun-shine about that Pole, it is always Day till the Sun decrease in Declination; for the Sun goes not below their Horizon, as you may see by turning the Globe about upon its Axis: And in the opposite Pole at the same distance the Sun-shine not reaching thither, it will be always Night till the Sun decrease in Declination, because the Sun riseth not above their Horizon.

5. If you let the Globe hang stiddy, you may see on the East-side of the Globe in what places it grows Night; and on the West-side of the Globe how by little and little the Sun encroaches upon it, and therefore there makes it Day.

6. If you make of Paper or Parchment a narrow Circle to begirt the Globe just in the Equinoctial, and divide it into 24 equal parts to represent the 24 Hours of Day and Night, and mark it in order from 1, 2, 3, to 12, and then begin again with 1, 2, 3,

to other 12, you may by placing one of the Twelves upon the Equinoctial under the Meridian of your place have a continual Sun-dial of it, and the Hour of the Day given on it at once in two places, to wit, where the enlightned Hemisphere is parted from the shadowed both on the East and West-side of the Globe; much more might be said on this subject, but the ingenious Artist may of himself find out diversities of other speculations.

And thus much shall suffice to have spoken concerning the use of the Cœlestial and Terrestrial Globes, and so I shall return from this Digression to my former-proposed Method; and shew you how most of those formerly mentioned Questions of the Sphere, or Spherical Problems, pag. 398, may be resolved.

S E C T. 2.

By Theakers Planisphere.

This Instrument consists of two Spheres or Plains, the lowermost and fixed, wherein is described three several Hour-Circles, the uppermost divided into 24 hours, wherein is reckon'd the Right Ascension of the Sun or Stars in Hours and Minutes: The next is called *Aries* Hour-Circle, having γ standing betwixt 24 of the outward Hour-Circle, and 12 in the inward or third hour-

hour-Circle ; in this is reckon'd the time of the Sun and Stars rising and setting, or of the Stars coming to the South : The third hour-Circle is divided into twice twelve, and in this is reckoned the difference of time, and next to this is the *Æquinoctial* Circle divided into 360 degrees: In the upper moving Sphere or Plain is drawn, First, the Circle of Months, placing *Aries* at the 10th day of *March*, by which means you may there find any day of the Month you desire : Secondly, Next to this is again placed another *Æquinoctial* Circle divided into 360 degrees. Thirdly, The *Ecliptick*, containing the 12 signs of the *Zodiack*, each sign divided into 30 degrees. There is also belonging to this moving Plain a moving Meridian, I call an Index graduated both ways from the *Æquinoctial*, whose length is equal to the Diameter of the *Æquinoctial* described on the upper moving Plain : The other parts whereof beyond what is graduated serves to cut the Circle of Months, the *Æquinoctial* and hour-Circles on the lower Plain as occasion is. Lastly, There ought to be also a turning Horizon which may be fitted for any Latitude, having two Indexes thereunto belonging, a longer and a shorter according to the Pattern, to make which Horizons for all Latitudes upon a piece of Past-board, draw

draw 2 lines perpendicularly crossing each other in the middle, the length of one of these Lines make equal to the Diameter of the upper Sphere ; and upon the other Line from the Center mark out the Latitude, by which means there is found three points, now find the Center to those 3 points, and strike an Arch, and cut out by that Arch the Horizon required.

P R O B L E M.

How by this Instrument to find the Declination place in the Ecliptick, and the Right Ascension of the Sun or any Star.

First, Turn the upper Sphere till *Aries* ♈, or the 10th of *March* stands directly against *Aries* ♈ in the lower Sphere. Then bring the moving Meridian to the Day of the Month, if your question be concerning the Sun ; or if it concern a Star, lay the Index upon the Center of the Star (whereof you find many with their Names added upon the upper moving Sphere) This done, observe that the Distance betwixt the *Æquinoctial* on the upper plain, and the Suns place or Star, is the Suns or Stars Declination ; and the Point or Place cut in the Ecliptick at the same time by the Index, shews the Sign, Degree, and Minute, the Sun or Star is

in; being their place or Longitude in the Ecliptick: And the Degree and Minute then cut in the Equinoctial by the Index, shews the Sun or Stars right Ascension in Degrees and Minutes, and the same is shewn in Hours and Minutes in the outermost Hour-Circle of the lower Plain: For Example, the 20th of *April* the Suns Declination is 15 degrees North, his place is 10 d. 8 m. of *Taurus*, his Right Ascension 38 d. which in time is two Hours, 32 minutes.

The 17th of *June* the great Dogs declination, 16 d. 32 m. his Place or Longitude 7 degrees of *Cancer* ☉: His Right Ascension 98 d. and in time 6 Hours, 32 Minutes.

P R O B L. 2.

To find the Rising and Setting of the Sun or any Star.

If the Sun or Star have North Declination, bring the longest Index of the Horizon to 12 Hours in *Aries* Hour-Circle, which will also cut 20 degrees of Longitude in the Equinoctial, the Meridian of *London*: But if the Sun or Star hath South Declination, then bring the shortest Index of the Horizon to the same place, and there keep the Horizontal Index fast, and if you desire to know its Rising, turn the upper moving

ving Sphere till the place of the Sun or the Star come exactly to touch the Circular Edge of the Moving Horizon in the East Semi-Circle of the moving Sphere; or if you desire its setting, bring the Suns place or Star to touch the Horizon in the Western Semi-Circle: Then turn the moving Index or Meridian to the Day of the Month, and the same Index will there shew you the time of the Sun or Stars rising or setting in *Aries* hour-Circle: For Example, the 20th of *April* the Suns place being 10 d. 12 m. of *Taurus*, riseth at 4 h. 48 m. in the morning, and sets at 7 h. 12 m. in the Afternoon. Also the great Dog the 10th of *March* riseth at One a Clock, 52 minutes in the Afternoon, and sets at 11 Hours, or 11 a Clock 8 minutes at Night.

P R O B L. 3.

To know when any Star comes to the South.

First, Bring the Moving Meridian to 12 hours in *Aries* Hour-Circle, and 20 degrees in the *Æquinoctial*, the Longitude or Meridian of *London*: keeping that fast, move the upper Sphere till the Star come just to the graduated edge of the moving Meridian, and there keep the Star fast, then turn the said moving Meridian to the Day of the Month,

Month, and in *Aries* Hour-Circle the same moving Meridian will shew you the time of that Stars coming to the South : So the great Dog the 1st of *January* comes to the South 11 a Clock at Night ; and *Orions* left shoulder the same day at 9 h. 37 m. &c.

To use Theakers Planisphere instead of a Nocturnal, or by it to find the Hour of the Night by any Star which is thereon.

This may be done two several ways : The first is the same with that described (376.) *Prob. 3.* for to know at what hour any Star comes to the South must necessarily shew what Hour it is when that Star comes to the South, and consequently any other hour may be found by knowing how many Hours such a Star is past the Meridian, or short of it : And therefore,

1. Suppose 12 Hours in *Aries* Hour-circle, and 20 d. in the Equinoctial, the Meridian of *London* (which are both one) to be instead of the *Flower de luce*, or North point in a Nocturnal : By help of the Index bring the Star on the Instrument so many hours past it, or short of it, as the same Star is past or short of the North point in Heaven : Keeping the Star to that position, move the Index to the Day of the Month, and in *Aries* Hour-Circle the same Index will shew you the Hour of the Night. 2. Sup-

2. Suppose γ to be instead of the *Flower de luce* or the North Point, bring the Star in a due position thereunto, and the day of the Month shall point exactly to the Hour of the Night in the third Hour-Circle, next below *Aries* Hour-Circle in the Fixed Plain: So the first of *April* the bright Star in the Rump of the great Bears Tail being upon the Meridian, either of these ways it will appear to be $\frac{1}{4}$ of an hour past 11.

P R O B L. 4.

To find at what time the Moon, or any of the Planets come to the South.

First, You must know the Sign and Degree in the Ecliptick which the Moon, or Planet is in; and bringing the moving Meridian or Index to 12 Hours and 20 degrees the Longitude of *London*, bring the degree which the Moon or Planet is in to the edge of the Meridian, and there keep both fast, and then look in the Circle of Months, and right against the day of the Month in *Aries* Hour-Circle, you will see at what a Clock that Planet comes to the South; but for the Moon you must remember to add 2 Minutes for each Hour, *vide Theaker's Book, pag. 20.*

Those that desire to know more of this Instrument,

Instrument, I refer them to the same Book of *Theaters*, and so pass on to shew how the most of these Questions of the Sphere may be found.

P R O B L. 5:

As an Appendix to these things, Concerning
the Moon, I shall shew you how to find the
Moons place.

Multiply the Moons Age by 2, and divide the Product by 5, the Quotient will shew how many Signs, and the Remainder or Fraction, so many times 6 degrees as the Moon is gone from that sign and degree where the Sun is at that present: Or more readily by this following Table.

[illegible]

The

The Days of the Moons Age.	D.	S.	D.	M.	The Hours of the Moons Age.	H.	S.	D.	M.
1	0	13	11		1	—	0	33	
2	0	26	21		2	—	1	6	
3	1	9	32		3	—	1	39	
4	1	22	42		4	—	2	12	
5	2	5	53		5	—	2	45	
6	2	19	3		6	—	3	18	
7	3	2	14		7	—	3	51	
8	3	15	25		8	—	4	24	
9	3	28	35		9	—	4	56	
10	4	11	46		10	—	5	29	
11	4	24	56		11	—	6	2	
12	5	8	7		12	—	6	35	
13	5	21	18		13	—	7	8	
14	6	4	28		14	—	7	41	
15	6	17	39		15	—	8	14	
16	7	00	49		16	—	8	47	
17	7	14	—		17	—	9	20	
18	7	27	11		18	—	9	53	
19	8	10	21		19	—	10	26	
20	8	23	32		20	—	10	59	
21	9	00	42		21	—	11	32	
22	9	19	53		22	—	12	5	
23	10	3	3		23	—	12	38	
24	10	16	14		24	—	13	11	
25	10	29	25						
26	11	12	35						
27	11	29	46						
28	00	8	56						
29	00	22	7						
30									

The use of the Table by an Example.

As suppose the 19th day of *June*, 1675, at Noon, you would know what sign the Moon is in: The change was the 12th day, at 8 Hours at Night; therefore the 19th day at Noon the Moon is 6 days 16 hours old.

The place of the Sun and Moon at the change was 3 signs, 1 degree, 0 *m.* the Moons motion for 6 days by the Table is 2 signs, 19 *d.* 3 *m.* and for 16 hours, 8 *d.* 47 *m.* the sum is 5 *s.* 28 *d.* 50 *m.* that is in 28 *d.* 50 *m.* of *Virgo*, which is the 5th sign from *Aries*: The like may be done at any other time. So much shall suffice to have spoken concerning the Moon.

S E C T. 3.

By the Quadrant.

I shall first give you a Description of the Lines thereupon, and then shew you the uses thereof: The double Arch next the Center is only for Ornament: The next Arch is the *Æquinoctial*, and the next to that represents the *Tropique*; betwixt which two Arches is the Line of Declination upon the side of the Quadrant, containing 23 *d.* 30 *m.* and within are the *Azimuth* or hour-lines: Also Crosswise betwixt the

the *Æquinoctial* and *Tropique* is the *Ecliptick*, containing three of the signs, each sign divided into 30 *d.* or at least supposed to be so: Next to the *Ecliptick*, and joyning with it in the *Æquinoctial* is the *Horizon*, divided, or supposed to be divided into 40 *d.* and ending or crossing the *Tropick* in 33 *d.* 9 *m.* Next below the *Tropick* is the *Circle of Months*, which you may know by the *Letters* prefixt: The next *Arch* below these is the *Quadrant*, or that which serves instead thereof, the *Line of right shadows*, and contrary *shadows* numbred from 1 to 10, and so back again to 1.

And the lowest of all is the *Limb* of the *Quadrant* divided into 90 degrees, and each degree into halves. For the use thereof;

1. The *Thread* laid on the day of the *Month*, and the *Bead* just to touch the *Hour-Line* of 12: at the same time the thread will shew in the *Limb* of the *Quadrant* the *Meridian Altitude* of the *Sun*, and moving the *Bead* from one *Hour* to another, will shew the *Sun's Altitude* at any *Hour* whereon the *Bead* resteth, and the *Bead* amongst the *Hour-Lines* will shew the *Hour of the Day*.

2. *How to find the Hour of the Night by some principal Stars set upon the Quadrant.*

How to place any Star upon the Quadrant,

First,

First, You must know the Stars Declination from the Æquator, and his Right Ascension from the next Æquinoctial Point: As the Declination of the Wing of *Pegasus*, being 13 d. 17 m. His right Ascension 358 d. 34 m. from the first point of *Aries*, or 11 d. 26 m. short of it: If you draw an occult parallel through 13 d. 17 m. And then lay a Ruler to the Center A, and 1 d. 26 m. in the Quadrant, the point where the Ruler crosseth the Parallel, shall be the place for the Wing of *Pegasus*, and so for the rest.

Stars.	R. Ascension	Declination.
Arcturus	30 d. 7 m.	21 d. 6 m.
Lyons Heart	32 d. 28 m.	13 d. 42 m.
Bulls Eye	64 d. 18 m.	15 d. 46 m.
Aquila	66 d. 26 m.	8 d. 33 m.

To find the Hour of the Night by these Stars.

First, put the Bead to the Star which you intend to observe, take his Altitude, and note how many Hours he is from the Meridian, then out of the right Ascension of the Star take the right Ascension of the Sun converted into Hours, and mark the Difference; for this difference being added to the observed Hour of the Star from the Meridian, shall shew how many Hours the Sun is gone from the Meridian, which in effect is the Hour of the Night: As for Example, the 9th of July the Sun being then in 25 d. 5

of

of *Cancer*, I set the Bead to *Oculus Tauri*, or the Bulls Eye, and observing his Altitude, should find him to be in the East about 12 degrees high, and the Bead to fall on the hour-line of 6 before noon, which is 18 hours past the Meridian, (or past 12 at Noon) The hour of the Night would be better than a Quarter past two of the Clock in the morning. For 118 *d.* the Right Ascension of the Sun converted into time, make 7 *h.* 52 *m.* this taken out of 4 *h.* 15 *m.* the Right Ascension of *Oculus Tauri* adding a whole Circle (or 24 hours) for otherwise there could be no subtraction, the Difference will be 20 *h.* 23 *m.* and this being added to 18 hours, which was the observed Distance of *Oculus Tauri* from the Meridian, shews that the Sun (abating 24 hours for the whole Circle) is 14 *h.* 23 *m.* past the Meridian, and therefore 23 *m.* past two of the Clock in the morning.

3. At what Hour the Bead toucheth the Line of Declination, that is, the Hour of the Suns rising or setting: And bring the Bead to the Horizon, and the Degrees cut in the Limb is the Ascensional Difference.

4. The Degrees cut by the Bead in the Line of Declination, shews the Suns Declination for that Day.

5. The Bead brought to the Ecliptick shews the Suns place.

6. Brought

5. Brought to the Horizon shews the Amplitude, which is always North when the Sun is in Northern Signs, and South when the Sun is in Southern.

6. The Bead shews the Azimuth, the thread being laid to the Suns Altitude.

7. The Thread laid to the Suns place, it shews in the Limb of the Quadrant his right Ascension.

8. Moving the Thread backwards from the Line of Declination towards the hour-line of 12 : It shews in the Limb the degrees of the Suns Depression under the Horizon at each hour, and consequently when it is 18 degrees under the Horizon it shews the beginning of day-break, and end of twilight.

Thus you see all these Astronomical Questions are easily and speedily resolved by the Quadrant, but observe that these Quadrants do serve only for some particular Latitudes.

As an Appendix I shall add the ingenious use of the Quadrant upon the Quadrant, and so conclude this Point : This which is called the Quadrant is supplied in a Line drawn next above the Limb of the Quadrant numbred from 1 to 10 in the middle, and so back again to 1, (as in the Description,) whereof the former part are called

S

Right

Right shadows, and the latter to the right hand are called contrary shadows, and by these you may find an Height by a known Distance, or a Distance by a known Height. And for performing hereof take these Rules.

1. If the thread fall in a right shadow, as 100 is to the parts on which the Thread falleth, so is the Distance to the Height required: Or, as the parts cut by the Thread is to 100, so is the Height to the Distance required.

2. If the Thread fall in a contrary shadow, as the Parts cut by the Thread are to 100, so is the known distance to the Height required: Or, as 100 is to the parts cut by the Thread, so is the known Height to the distance required. To make this more plain, if the thread fall on 100 parts, which is denoted by 10 in the Quadrant, or on 45 degrees in the Quadrant; the Height is equal to the distance above the Level of your Eye: If it fall on 50 parts of Right shadow, the Height is half the distance, if it fall on 25, it is 3 quarters of the distance: But if the Thread fall on 50 parts of contrary shadow, the Height is double to the distance; If on 25, it is 4 times the distance, and the like of the Rest.

C H A P. VIII.

BUT to return to my proposed Method, having shewn you how these Sphærical Problems or Questions of the Sphear may be easily and readily resolved by those Artificial Projections of the *Globe*, the *Plani-sphere*, and *Quadrant*, and by way of Digression, having added the other uses also of these Instruments, I come now in the last place to shew how these Questions may be resolved by *Projection of the Sphere in plano*. And to do this,

1. I shall shew you how to *Project the Sphere in plano*.

2. How these Sphærical Triangles do naturally lye in the Sphere it self: Thus Projected.

3. How the Sides and Angles of these Sphærical Triangles may be measured in the Projection by a Line of Cords only, whereby these Questions are most ingeniously resolved.

S E C T. 1.

How to Project the Sphere in plano upon the Plain of the Meridian.

I might shew you several ways of Pro-
jection

jection, as that which they call the Analemma, which is by drawing straight lines from one side of the Circle to the other, and thus you will find it projected by Mr. Norwood in his *Seamans Companion*, and in *Brown*, pag. 40. Also in Mr. Gunter very ingeniously: But that which is most generally used is the Stereographick Projection, used by *Gemma Frisius*, which you will also find in *Brown*, p. 142, and in *Philips Geometrical Seaman*, pag. 75, which is the way used in Maps of the whole World: But that which includes all the rest in some respect, and which is most suitable to my present purpose, is that Projection of Mr. *Leyburns* in his *Geometrical Exercises*, pag. 122. which I here follow, being to be done by a Line of Chords only, and upon which Projection by the interfection or crossing of the several Circles thereof, are constituted divers Spherical Triangles: By resolving of which, most of these fore-mentioned Astronomical Questions of the Sphere may with speed and exactness, as also with much pleasure and delight be resolved.

First then, Take 60 Degrees or Radius from your Line of Chords, and with that distance upon the point A, as a Center describe the Circle Z H N O, representing the Meridian (within which all the rest are to be projected) and cross it with the two
Diame-

Diameters HAO, and ZAN, the former representing the Horizon, the latter the Zenith and Nadir, or the prime Vertical.

Secondly, (Because the Latitude of the place for which you draw your projection is $51^{\circ} 30'$.) Take $51^{\circ} 30'$ from your Line of Chords, and set them upon the Meridian from O to P, and from H to S, and draw the Line PAS, the Axis of the World P signifying the North Pole, and S the South Pole: Also with the same distance set off $51^{\circ} 30'$ from Z to $\mathcal{A}$, and from N to $\mathfrak{a}$, and draw the line $\mathcal{A}A\mathfrak{a}$ for the Equinoctial.

Thirdly, Take $23^{\circ} 30'$ the Suns greatest Declination, and also the Distance of the 2 Tropicks from the Equinoctial, and set them upon the Meridian from $\mathcal{A}$ to $\mathfrak{S}$, and from $\mathcal{A}$ to $\mathfrak{V}$; and also from $\mathfrak{a}$ to $\mathfrak{S}$, and from $\mathfrak{a}$ to $\mathfrak{W}$: This done, draw a right Line $\mathfrak{S} A \mathfrak{W}$ between the two Tropicks, touching the Tropick of *Cancer* above the Horizon, and the Tropick of *Capricorn* below the Horizon, and this is the Ecliptick: Then take $23^{\circ} 30'$ out of your Line of Chords, and set them from P to Q, and from S to R, and draw the Line QR the Axis or Poles of the Ecliptick. Now to divide the Ecliptick into signs and degrees, take 60° from your Line of Chords, and set them from Q to 1, and from Q to 3: Also set the same distance

from $\mathfrak{S}$ to 2, and from $\mathfrak{W}$ to 4; this done, lay a Ruler to the Pole R, and the figure 1, it will cut the Ecliptick in the point $\mathfrak{U}$ and $\mathfrak{Q}$: The Ruler laid to R and 2 will cut it in the points $\mathfrak{V}$ and $\mathfrak{X}$, and laid to R and 4 in $\mathfrak{M}$ and $\mathfrak{K}$; and laid to R and 3 in $\mathfrak{T}$ and $\mathfrak{Z}$. And if you divide every of the spaces $\mathfrak{S}$, 1, 12, 2 Q, Q4, 4 3, and 3 $\mathfrak{W}$ into 3 equal parts, each part will contain 10 degrees, and a Ruler laid to each of them, and the point R shall give you the points upon the Ecliptick answering to the 10 degrees of every sign, and in the same manner you may put on every degree. Thus you have drawn the Meridian Horizon, the prime Vertical, the Axis of the World, and the $\mathfrak{A}$ equinoctial, the Axis of the Ecliptick, and the Ecliptick, all which are streight Lines: Now we come to draw those which are Circular, or Arches of Circles; and before you can draw these, the Centers of those Circles must be found out: Now I shall shew you two ways of finding out these Centers: One of these is Mr. *Phillips* in his *Geometrical Seaman*, on this manner, divide the Meridian into degrees in 4 Quadrants, and keeping one end of your Ruler fixed at the Point at Z: Lay the other end to the several degrees in the lower Semi-circle HNO, so you shall divide the Diameter HO into 9 parts, which are half-

half Tangents. In the same manner, if you keep one end of your Ruler, fixed at the point H, and lay the other end to the several degrees of the Semi-Circle ZON. You may divide the Diameter ZN into 9 unequal parts, which are half-Tangents. Now having thus divided the Diameters, they must guide you in drawing of the Meridians, and Parallels which are all parts of perfect Circles: And you may find their Centers by these three points. First, for the Meridians, they all concur in both the Poles, and their third point is their correspondent degree in the opposite Diameter, and for the Parallels, two of their points are their degrees in the outward Circle, and their third point is their correspondent degree in the opposite Diameter: Further, you must know that the Centers do all lye in the Diameter Lines, extended beyond the Circle, (if you seek for the Center of a Meridian exceeding 45 degrees) for the Diameter being divided into half-Tangents, if for every degree you account 2, beginning from the Center A, so you shall have the Centers of the Meridians, and if you set one foot of your Compasses in that Center, and open the other to the Point Z or N, it will pass through the Correspondent degree in the Diameter H. O. As for Example, if you would draw the Meridian

ZGN, which cuts the Diameter in 32 degrees, HG, count from the Center towards O, double so many degrees as is betwixt H and G, which is 64 degrees, and that is the Center to the Meridian, Z. G. N. viz. W. The like is to be done for finding the Centers to the Parallels, as you may easily apprehend: But if any of these Centers fall so far without the Circle, that your Compasses will not reach them, there is an Instrument call'd a Bow, which by help of one or more screws may be extended to touch any three points which lye near in a straight Line; or a Ruler bridled may do the same if it bend equally.

This way of Mr. *Phillips* finding the Centers I cannot but applaud as very ingenious, but do prefer that of Mr. *Leyburns*, which is thus: Upon a long piece of stiff paper describe a Quadrant as ABC, and upon the point C erect an exact perpendicular. Let the Quadrant be divided into 90 degrees, and consider how many degrees the Arch of the Circle (whose Center you seek) is distant from the Meridian or outward Circle, and a Ruler laid from the Center of the Quadrant by so many degrees in the Limb thereof to cut the perpendicular, where the Ruler cuts the perpendicular make a prick or mark, and the distance betwixt the said mark and the Center of the Quadrant A shall

shall be the Semidiameter of the Circle sought: To make this plain by these following Examples, First, for the Tropicks, whose Distance from the Equinoctial is $23^d. 30^m.$ lay the Ruler to the Center of the Quadrant A, so as it cut $23^d. 30^m.$ from the point B in the Limb at *f*, and cuts the perpendicular at *g*, the Distance A *g*, or the Line A *f g* taken in your Compasses, and setting one foot upon the Polar Line extended till the other foot just reach the Equinoctial at $\text{\AA}$; draw an Arch of a Circle, and it shall be one of the Tropicks, as $\text{\textcircled{S}} \text{ I } \text{\textcircled{S}}$ the Tropick of *Cancer*, and $\text{\textcircled{W}} \text{ k } \text{\textcircled{W}}$ the Tropick of *Capricorn*: The like may be done by any other parallel of Declination, or Latitude, whether they fall nearer or further off the Equinoctial.

Another Example take in the Hour-Circle PBS, which is distant from the Meridian $62^d. 46^m.$ wherefore take $62^d. 46^m.$ from your Line of Chords, and set them from *a* to *b*, then laying a Ruler from P to *b*, it will cut the Equinoctial in B, through which point the hour-Circle of $62^d. 46^m.$ must pass. To find the Center belonging to this hour-Circle, lay your Ruler to the Center of the Quadrant at A, and let it pass by $62^d. 46^m.$ in the Limb: From the point C it will cut the perpendicular in M, therefore taking the Distance in A in your

Compasses, and setting one foot in the Equinoctial extended, and with the other jult to reach P or S, strike an Arch of a Circle, and it will cut PBS the Hour-Circle of $62^{\circ} 46'$. The like may be done in any other Hour-Circle, or Azimuth-Circle, Parallels of Declination, Circles of Altitude, or any other whatsoever; only observe, that the Centers of all Hour-Circles fall in the Equinoctial Line $\text{Æ A } \alpha$ extended, the Centers of Azimuth-Circles in the Horizon HAO extended, where need is: The Centers of the Tropicks, and Parallels in the Axis of the World PAS, and the Centers of the Circles of Altitude fall in the prime Vertical Circle ZAN.

Lastly, As every Circle in the Projection hath its proper Center, so hath it also its proper Pole. For the finding whereof, note that the Pole of a great Circle is 90° degrees, or a Quadrant distant from the Circle it self upon that Line which cutteth the Circle at Right Angles: Thus the Poles of all the Hour-Circles are upon the Equinoctial: The Poles of all the Azimuth Circles upon the Horizon, &c. Now if you would know the Pole of the Hour-Circle PDS, lay a Ruler upon P and D, and it will cut the Meridian-Circle in e : Then take 90° degrees from your Line of Chords, and set them from e to f , a Ruler laid from P

to *f*, will cut the Equinoctial in *Y*, so is *Y* the Pole of the Hour-Circle *PDS*, the same Rule is to be observed in finding the Poles of the Azimuth-Circles: As if you would find the Pole of *ZGN*, lay a Ruler upon *Z* and *G*, it will cut the Meridian Circle in *g*, then set 90 degrees of your Chord from *g* to *d*, so a Ruler laid from *Z* and *d* will cut the Horizon *HAO* in the point $\odot$, which is the Pole of the Azimuth-Circle *ZGN*; and by the same manner of work you may find the Poles of all the rest. As of *PDS* is *Y*, *PCS*, *V*; *PAS*, $\text{\AA}$ or *a*, *PBS*, *ZGN*, $\odot$; *ZAN*, *H* or *O*; *ZFN*, *T*; *ZON*, *G*; *HAO*, *Z* and *N*; $\text{\AA}$ *A* *a*, *P* and *S*; $\text{\AA}$ *A* $\text{\AA}$, *Q* and *R*; And thus much shall serve to shew you how to Project the Sphere *in plano*, and how to find the Centers and Poles of all the Circles.

S E C T. 2.

In the next place I come to shew you how the forementioned Questions of the Sphere may be speedily and exactly resolved by the Resolution of several Spherical Triangles which naturally lye within the Sphere being thus projected: And of these in this following order: Having before shewn you how to measure the side or Angle of any Spherical Triangle, *pag.* 344, 345.

PROP.

P R O P. 1.

The Distance of the Sun from the nearest Equinoctial Point (either Aries or Libra) given, to find his Declination.

In the Projection this Proposition is to be resolved upon the Triangle $Ak\approx$; where-
in you have given, 1. The Side $A\approx 59 d.$ the distance of the Sun from the nearest Equinoctial point *Libra*. 2. The Angle $\approx Ak$, $23 d. 30 m.$ which is the Angle that the Ecliptick makes with the Equinoctial, and is always equal to the greatest Declination of the Sun, and you are to find the side $k\approx$, which to do by the Projection.

Lay a Ruler upon the Pole of the Circle $R\approx Q$ (which is at $\varnothing$) and the point k . it will cut the Meridian in the point l : Also lay a Ruler from $\varnothing$ to $\approx$, it will cut the Meridian in the point w , so the distance $l w$ being measured upon your Line of Chords, will contain 20 degrees, the Suns Declination, he being in $29 d.$ of $\approx$.

P R O P. 2.

To find the Latitude, or the Side PO of the Triangle $l\odot O$.

You need do no more when you are to measure any side of a Triangle, which lies
in

in the Meridian it self, than to take the same side in your Compasses and meature it upon your Line of Chords, as in this Proposition. Take in your Compasses the Distance from O to P, and measure it upon your Line of Chords, and it will be found to contain 51 d. 30 m. the Latitude required.

P R O P. 3.

The Latitude of the place, and Declination of the Sun given, to find his Amplitude.

This Proposition may be resolved by two several Triangles, one is $P \odot O$, the other $A \odot B$. In the first you have given the side PO the Latitude, and $P \odot$ the Complement of the Suns declination, and you are to find $\odot O$. To do this by the Triangle, $A \odot B$, lay a Ruler upon Z, (which is one of the Poles of the Horizon) and to the point $\odot$ it will cut the Meridian Circle in d, and laid from Z to A, it will cut the Nadir point in N; so the distance N d will be found 33 d. 20 m. The Amplitude of the Suns Rising or Setting from the true East or West points of the Horizon. In the Triangle $A \odot B$, you have given $\odot B$ the Suns Declination 20 d. The Angle $\odot AB$, the Complement of the Latitude 38 d. 30 m. and you are to find the side $\odot A$ as above.

For

For finding the side $\odot O$ in the Triangle $P \odot O$, lay a Ruler to Z , the Pole of the Horizon, and the point $\odot$, and it will cut the Meridian Circle in D , so the distance dO measured upon your Line of Chords, will contain 56 d. 40 m. which is the Amplitude of the Suns Rising or Setting from the North point of the Horizon.

P R O P. 4.

The Suns greatest Declination, with his Distance from the next Equinoctial point being given, to find his Right Ascension.

The Triangle $A k \approx$ resolves this proposition, wherein you have given, 1. The side $A \approx$ the distance of the Sun from *Libra*. 2. The Angle $\approx Ak$ the Suns greatest Declination, and the right Angle at $\approx$, and you are to find Ak the side of the Triangle, Lay a Ruler to P and k , it will cut the Meridian in s , and lay it again to P and A , it cuts the same in S , so the Distance Ss will be found 56 d. 50 m. the Suns Right Ascension.

P R O P. 5.

The Latitude and Declination given, to find the Ascensional Difference.

This is resolved in the Triangle $A \odot B$, wherein you have given, 1. The side $\odot B$,
the

the Suns Declination 20 *d.* 2. The Angle $\odot$ AB, the Complement of the Latitude 38 *d.* 30 *m.* and the Right Angle at B, and You are to find the side AB. Lay a Ruler to P, the Pole of the World (and also of the Equinoctial) and the point B, it will cut the Meridian Circle in the point *b*, the distance *b. s.* measured, will be 27 *d.* 14 *m.* the Ascensional difference, which is so much as the Sun Riseth or Setteth before or after six a Clock.

P R O B L. 6.

The Latitude, Declination, and Altitude of the Sun given, To find the Suns Azimuth either from the East, North, or South points of the Horizon.

That which is most difficult and intricate to perform by Numbers, is by Projection effected with the same ease as any of the rest: As in this proposition it is the Angle EZP, in the Triangle ZEP, which is here required: Lay a Ruler upon Z to the point G upon the Horizon, the Ruler thus laid, will cut the Meridian in G, so the distance *g. O.* measured upon the Line of Chords, will be 146 *d.* which is the Suns Azimuth from O the North part of the Meridian, and from *g* to H 34 *d.* which is the Azimuth from H the South-part of the Meridian,
and

and from g to N 56^d . which is the Azimuth from A , the East and West point of the Horizon.

P R O P. 7.

The Latitude, Declination, and Altitude of the Sun given, to find the hour of the day.

This Proposition is performed by resolving the Triangle $Z P a$, where you are to find the Angle $Z P a$: Wherefore lay a Ruler from the point P , to the point a , and it will cut the Meridian in t , so the Arch $t \text{ } \text{AE}$ will be found $95^d. 52^m$. which is the hour from the Meridian: The Arch $t a$ being measured, will contain $84^d. 8^m$. which is the hour from midnight: Also the Arch $t S$ being measured, will contain $5^d. 52^m$. the hour from six.

This Triangle $Z P a$ is composed of $Z P$ an Arch of the Meridian $Z a$, an Arch of an Azimuth Circle, and of $P a$ the Arch of an hour-Circle.

P R O P. 8.

The Declination, Altitude and Azimuth of the Sun given, to find the hour of the day.

The Triangle ZEP in the projection resolves this Problem, wherein there is given

1. EP ,

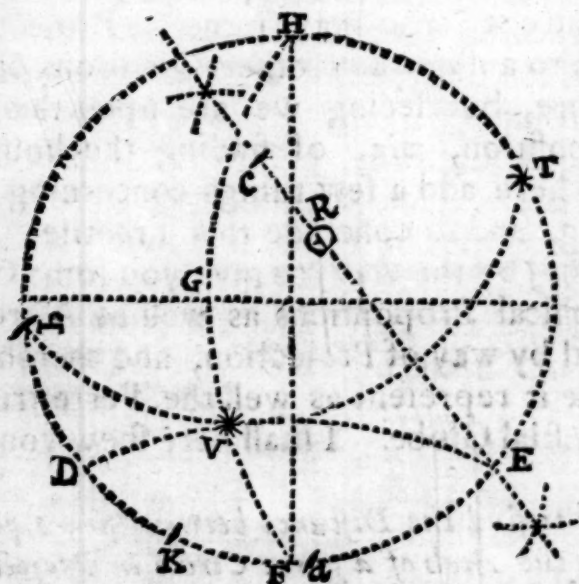
1. EP, the complement of the Suns Declination 70 *d.* South. 2. The side EZ the complement of the Suns Altitude 78 *d.* And 3. The Angle E郑 the Suns Azimuth from the North. The Angle ZPE is to be found to answer this proposition: Therefore lay a Ruler to the Pole P, and upon the point D in the *Æquinoctial*, and it will cut the Meridian in *e*. The distance *e* *Æ* being measured, will give you 35 *d.* 36 *m.* the hours from Noon which in time is 2 *h.* 22 *m.* Many more propositions might be added, which I leave to the ingenious practitioner to find out; who may frame his Projections so as to answer any other Questions of the Sphere, but seeing we are upon this last Proposition, *viz.* of finding the hour, I shall here add a few things concerning Dialling, and so conclude this Treatise.

But (by the way) to give you some Geographical Propositions as well as Astronomical by way of Projection, and thereby to make it represent as well the Terrestrial as Cœlestial Globe. I shall here shew you,

How to find the Distance betwixt any 2 places by the Arch of a great Circle in Projection, howsoever distant from one another.

1. Draw a Circle, which may be supposed

fed to be the first Meridian, (which is reckoned from the Islands of Azores) cross this Circle through the Center with a Diameter, which is to be supposed the Equinoctial; if the two places differ only in Longitude, the Distance is to be measured upon the Equinoctial, or some other parallel: If they differ in Latitude only they are to be measured upon a Meridian; if they differ both in Longitude and Latitude, the Distance must be measured by drawing an Arch of a great Circle to pass through



both the places: And to measure any of those Distances, you must find the Pole to the great Circle which passes through both places:

places: How to do this, you have it elsewhere taught *pag.* 345: Then laying a Ruler to the said Pole, and first to one of the places, and afterwards to the other, and marking at each time where the Ruler cuts the Meridian or outward Circle: The same distance taken in your Compasses, and measured upon your Line of Chords, gives in Degrees and Minutes the Distance betwixt the two places, which you know being multiplied by 20, brings it into Leagues, or by 60 into Miles: I shall only give one Example for all: Suppose you would know the Distance betwixt the *Cape of Good Hope*, lying in the Latitude of 40 *d.* South, and in Longitude 50 *d.* and *Malibrigo* lying in 26 *d.* of North Latitude, and in 180 *d.* of Longitude you will find their distance to be 140 *d.* or 8400 miles by Projection thus, (H. T. E. F. K. D.)

First, draw the Circle, and from the same Line of Chords by which you draw the Circle take 26 *d.* for the Latitude of *Malibrigo*, and set it from *a* upwards, and because the Longitude is 180 *d.* there you must place it in the Meridian at T. which signifies *Malibrigo*. Now to place the *Cape of Good Hope* according to its Latitude and Longitude, do thus: Draw a Parallel of Latitude of 40 *d.* viz. DE, and a Meridian of 50 *d.* (H.G.F.) for the Longitude

gitude where these 2 intersect each other, is the place you design (the *Cape of good Hope*) there make a mark at V, and with the same Radius wherewith you drew the first Circle, find a Center which will cut T and V, which you may easily do by drawing a Line to pass exactly in the middle betwixt T and V, when that is done, you must find the Pole of the same Circle at R, and laying a Ruler from the Pole R to V, it will cut the Meridian in K. Lastly, Measure the Distance T K, and that gives you the degrees that one of the places is distant from the other.

Those that desire to be informed more at large concerning finding these Distances, I refer them to Mr. *Leyburn*, his Eighth Geometrical Exercise, pag. 131.

What is here said concerning finding these Distances, the ingenious may apply to Sailing by the Arch of a great Circle; but I shall leave that to the Readers own Study and Improvement.

To know how many Miles any place is distant from you, in whose Zenith the Sun, or any Star is at the same time.

Take the Altitude of the Sun, which suppose 53 d. from your Zenith; and with Mr. *Norwood* account 69 Miles in a degree, multiply 53 by 69 makes 3657, and so many miles

Miles it is between the place where I now am, and the place in whose Zenith the Sun was at that time, and by this means you may know how far it is to any place over, which the Sun or Star is at any time.

C H A P. IX.
Of Dialling.

THE whole and compleat Art of *Dialling* is a subject sufficient for a Treatise alone, and you will find several Authors have writ whole Books concerning the same, I shall confine my self within the compass of these few leaves which are left me; but what shall be said I hope will be plain and perspicuous to any ordinary Capacity: Brevity, and withal perspicuity being the Mark I aim at all along in this little Treatise.

There are many Denominations and Distinctions given to Dials, as *Horizontal, Vertical, Polar, Equinoctial; Erect, Direct, Inclining, Declining, and Reclining*, every way East, West, North, and South, and many more, both Fixed and Pendulant: Plain and Globular within doors and without: It is not my intention to speak to these particularly, but shall only shew you how to make an *Horizontal*, and an *Erect, Direct South Dial*, which are most usual, and most useful.

SECT.

S E C T. I.

To draw an Horizontal Dyal by the Globe.

I have shewed you pag. 374, how to make the Globe it self a Dyal, and by consequence any round Ball : I shall now shew you how to make an *Horizontal Dyal* by the Globe.

First then, prepare your flat Dyal-ground parallel to the Horizon, and thereupon draw a Meridian, which must represent the direct North and South, and cross the same at right Angles, which must represent the East and West. The first of which Lines is the 12 a Clock Line, the latter shews 6 a Clock both Morning and Evening; then making the Interfection of these Lines the Center, describe a Circle on the Dyal-ground to what distance you please, and divide the same into 360 d. (as all other Circles are) and subdivide the same into as many lesser parts as you can : Being thus prepared, you are to find the distances of the Hour-Lines in this Circle for any Latitude of place, which to do, let the Globe be set to the Latitude designed, and then make choice of some of the great Circles in the Globe that pass through the Poles of the World, as for Example (the *Æquinoctial* colure) and apply the same to the *Brazen Meridian*, in which scituation it represents the Substile or 12 a Clock Line, or Me-

ridian

ridian Line (call it which you will) then turning about the Globe towards the West, till 15 degrees of the Æquator have passed through the Brazen Meridian: You must mark the Degree of the Horizon which the same Æquinoctial colure crosses, for that point will shew the Distance of 1 and 11 a Clock from the Meridian or 12 a Clock-Line: Then turning again the Globe forwards till other 15 degrees of the Æquator do pass the Meridian, the same colure will point out the distance in the Horizon of the hours 10 and 2 from the hour of 12, and in the same manner you may find out the distances of all the rest of the hour-lines from 12 in the Horizon; but note that the beginning of these distances must be accounted from that part of the Horizon on which the Pole is elevated; Now these Distances of the hours being thus noted in the Horizon of the Globe, you must afterwards translate them into your plain allotted for your Dial-ground, reckoning in the circumference of it so many degrees to each hour as are answerable to those pointed out by the colure in the Horizon: Lastly, having thus done, the Gnomon or Style must be erected, where note that the edge of the Gnomon, which is to shew the hours by its shadow in all kind of Dyals, must be set parallel to the Axis of the World: That so it may make

an Angle of Inclination with its plain ground equal to that which the Axis of the World makes with the Horizon, and that the Style is to stand directly North and South, and in the Meridian Line is a thing so commonly known, that it were needless to mention it.

Now if you would make an Erect South Dial, know, That which is an Horizontal Dial in one place, will be an Erect South-Dial in another: As suppose you would make an Erect Dial for the Latitude of 52° : This is nothing else, but to make an Horizontal Dial for the Latitude of 38° . And if you make an Erect Dial for the Latitude of 27° . the same will be an Horizontal Dial for the Latitude of 63° . and the same proportion is to be observed in the rest; and hence it appears that an Horizontal Dial and a Vertical are the same at the Latitudes of 45° . But thus much shall suffice to have spoken concerning drawing Hour-Lines by the Globe in an Horizontal or an Erect South-Dial.

S E C T. 2.

The Analogy or Propositions to find them by are these.

P R O P. 1.

For an Horizontal-Dial.

As Radius or the sine of 90° . is to the sine of the Latitude (suppose 51° . 30^m .)

So

So is the Tangent of the hour, (*viz.* 15 *d.* which makes an hour in the Equinoctial.) To the Tangent of the Hour-Line from the Meridian, (*viz.* the distance of the hour of 1 from 12) which in this Example is 11 *d.* 51 *m.*; and so is 30 *d.* the Tangent of two Equinoctial hours to the Tangent of the distance from 12 to 2, &c.

P R O P. 2.

For an Erect-South Dial.

As the Radius (90 *d.*) to the Co-sine of the Elevation (in this Example 38 *d.* 30 *m.*) So is the Tangent of any hour given, (suppose 15 *d.* for the hours 11 and 1, or 30 *d.* for 10 and 2.) to the Tangent of the hour-Line, (or to the true Distance of the hour-line) from the Meridian, (which in this Example is 9. 28, and 19. 46, &c.)

S E C T. 3.

Here follows a Table taken out of *Seller*, shewing the distances of the Hour-Lines from the Meridian in several degrees of Latitude, both for an Horizontal, and also a South Erect Dial, as in the Latitude 54 the distance from 12 to 1 is 12 *d.* 41 *m.* from 1 to 2 is 25 *d.* 2 *m.* and so of the rest in an Horizontal plain: But in an South Erect Dial the Distance from 12 to 1 is 8. 56 from 1 to 2, is 18. 45, and so in any other Latitude you will find the degrees answering to each hour, which is very ready

T

dy

dy. The Style or Cock of the Dyal must be fixed just over the Meridian Line, and must make an Angle with the Plain equal to the Height of the Pole, the hours before 6 in the morning, and after six in the Evening, may be supplied by their opposite hours on the other side the Center.

A Table shewing the Distances of the Hour-Lines from the Meridian in these Degrees of Latitude.

Lat	50.	51.	52.	53.	54.	55.	56.	57.	58.	59.	H.
H.	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.	H.
—	2.52	2.54	2.56	2.59	3. 1	3. 4	3.07	3. 8	3.10	3.12	—
1.	5.44	5.50	5.55	6.00	6. 4	6. 9	6.13	6.18	6.22	6.28	11
2.	8.39	8.47	8.54	9. 1	9.10	9.15	9.21	9.27	9.34	9.43	10
3.	11.36	11.45	11.55	12. 5	12.13	12.23	12.32	12.39	12.40	13.00	9
4.	14.33	14.46	14.57	15. 8	15.19	15.31	15.42	15.52	16.02	16.16	8
5.	17.36	17.51	18. 3	18.17	18.29	18.45	18.57	19.08	19.21	19.38	
6.	20.39	20.56	21.11	21.27	21.40	21.57	22.12	22.24	22.38	22.58	
7.	23.51	24. 9	24.26	24.44	24.59	25.18	25.33	25.49	26.04	26.26	
8.	27. 4	27.23	27.43	28. 3	28.19	28.39	28.56	29.12	29.29	29.53	
9.	30.23	30.44	31. 7	31.26	31.44	32.05	32.23	32.41	32.59	33.24	
10.	33.51	34.12	34.34	34.57	35.15	35.39	35.59	36.16	36.34	37.00	
11.	37.27	37.50	38.13	38.36	39. 3	39.18	39.38	40.00	40.17	40.44	
12.	41.10	41.32	41.57	42.19	42.39	43.03	43.23	43.42	44.02	44.30	
13.	44.58	45.21	45.45	46.10	46.30	46.52	47.12	47.33	47.52	48.18	
14.	48.54	49.18	49.41	50.05	50.25	50.48	51.08	51.27	51.45	52.12	
15.	52.58	53.22	53.44	54. 7	54.26	54.47	55.06	55.25	55.43	56.09	

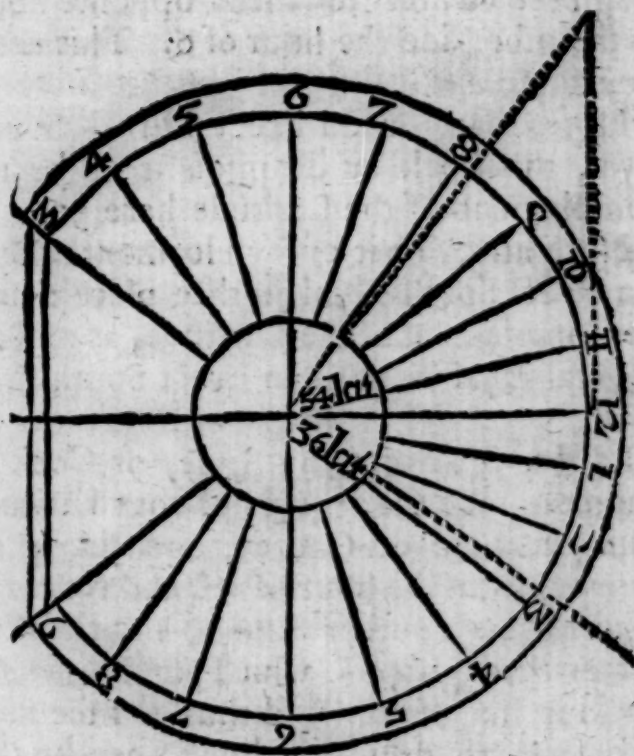
	50.	51.	52.	53.	54.	55.	56.	57.	58.	59.	
H.	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.	H.
5	57.13	57.35	57.56	58.17	58.35	58.56	59.14	59.31	59.48	00.22	7.
	61.33	61.53	62.13	62.26	62.48	63.07	63.23	63.39	63.55	64.17	
	66.	55.62	66.40	66.57	67.11	67.28	67.42	67.56	68.10	68.29	
5	70.41	70.56	71.09	71.25	71.37	71.51	72.02	72.14	72.25	72.41	6.
	75.26	75.37	75.49	76.00	76.09	76.20	76.29	76.35	76.47	76.59	
	80.14	80.23	80.30	80.38	80.44	80.52	80.58	81.04	81.10	81.18	
	85.	28.06	85.10	85.14	85.18	85.22	85.26	85.31	85.34	85.39	
6	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	
	40.	39.	38.	37.	36.	35.	34.	33.	32.	31.	

These Tables of Mr. Sellars are for an Horizontal Dial, for Latitude 34 to 40, and from 50 to 59: The Complement whereof is for an Erect Direct South-Dial in such places of Latitude as is the Complement thereunto, as from 31 to 40, and from 50 to 56. This called by *Sellar* Tabular Dyalling, easiest of any other.

South.	56.	55.	54.	53.	52.	51.	50.	
Horiz.	34.	35.	36.	37.	38.	39.	40.	
	2.05	2.09	2.12	2.15	2.18	2.21	2.24	
	4.12	4.19	4.25	4.31	4.38	4.43	4.50	
	6.20	6.30	6.40	6.49	6.58	7.07	7.17	
1	8.31	8.44	8.57	9.09	9.22	9.34	9.47	11
	10.43	11.00	11.16	11.31	11.47	12.01	12.18	
	13.2	13.21	13.41	14.19	14.35	14.55	15.00	
	15.22	15.46	16.8	16.29	16.51	17.11	17.33	
2	17.52	18.18	18.44	19.8	19.33	19.56	20.21	10
	20.27	20.56	21.24	21.52	22.19	22.45	23.13	
	23.9	23.41	24.14	24.43	25.14	25.41	26.12	
	26.03	26.38	27.13	27.46	28.18	28.49	29.23	
3	29.11	29.49	30.27	31.05	31.36	32.08	32.44	9
	32.31	33.11	33.51	34.27	35.04	35.38	36.16	
	36.04	36.46	37.28	38.06	38.45	39.19	39.58	
	39.55	40.36	41.22	42.00	42.40	43.16	43.55	
4	44.03	44.48	45.29	46.10	46.48	47.24	48.04	8
	48.34	49.18	50.00	50.39	51.17	51.52	52.30	
	53.28	54.05	54.46	55.24	56.00	56.32	57.10	
	58.42	59.21	59.59	60.33	61.06	61.36	62.09	
5	64.20	64.53	65.27	65.56	66.26	66.52	67.20	7
	70.23	70.51	71.17	71.36	72.05	72.25	72.48	
	76.44	77.3	77.22	77.38	77.38	78.09	78.25	
6	83.18	83.28	83.38	83.47	83.55	84.03	84.11	
	90.00	90.00	90.00	90.00	90.00	90.00	90.00	6

To draw an Horizontal Dial by this Table for 54 d. Latitude.

First, Take with your Compasses from a Line of Chords 60 *d.* with that distance describe a Circle, cross that Circle thorough the Center with a Diameter for a Meridian Line, and cross that at right Angles for the hour of 6, then consider for what Lati-



tude you draw the Dial, which in this is 54, look for 54 towards the left hand of this

this Table, and over-against the Latitude, you will find 12 *d.* 14. for the distance of 11, and 1 from the Meridian, 25 *d.* 2 *m.* for the distance of the hours 10 and 2, &c. These you are with your Compasses to take from your Line of Chords, and prick them upon the Dyal-Circle, and so draw Lines to each mark, which are the hour-lines. After you come at the hour of 6 you may draw the remaining hours, by laying a Ruler thorough the Center to their opposite hours on the other side the hour of 6. This needs no Example.

Note, that for an Erect Direct South-Dyal, the Cock or Gnomon must be the Complement of the Latitude here 36.

Note also, That every Horizontal Dyal is a direct South-Dyal in that place that is the Complement of the Latitude, as an Horizontal Dyal in 52 is an Erect South Dyal in 38.

Lastly, To draw the Style, or Cock or Gnomon, take the degrees of your Latitude from the Line of Chords, and set it off sideways from the hour of 12, and from the Center draw another Line to intersect the former through 54 *d.* which you see is near the hour line of 8. And thus you see how easie it is to draw the hour-lines by this Table for an Horizontal: The same may be done for an Erect South-Dyal, if you
look

look for the Latitude on that side of the Table towards the right hand ; there against 54 you find 8 d. 56 m. for 11. and 1, and 18.45 for 10 and 2, and so forwards ; whereby you may understand to make an Erect South Dial for any Latitude mentioned in the Table : If you please to do it by Arithmetical operation, you have also the Analogy or Proportions to work them by ; or if you please to do it by the Globe, you may find the hour-lines by that also as is shewed in the former page, 407. And this is all at present I intend to say concerning finding the hour of the Day by the Sun, or making Sun-Dyals ; I shall only shew you how to find the hour of the Night by the Moon and Stars, and so bid good Night, and conclude this Mathematical Manual.

S E C T. 4.

First then, By the Moon.

You must understand, that by knowing the Prime you may find the Epact, and by knowing the Epact you may find the Age of the Moon ; by knowing the Age of the Moon you may find the time of her coming to the South ; and by knowing the time of her coming to the South, you may find the hour of the Night by the shadow of the Moon upon a Sun-Dyal. Therefore of these in order.

T 4

P R O P.

P R O P. 1.

To find the Prime or Golden Number,
Divide the year of our Lord by 19, and to
that which remaineth after the division add
1, the Product is the Prime Number for all
that year, beginning the 1st January: As
1673 [88 $\frac{1}{9}$ to $\frac{1}{2}$ 2 is the Prime.

199

P R O P. 2.

By the Prime, To find the Epact.

Multiply the Prime of the year by 11,
and Divide the Product by 30, and the Re-
mainder or Fraction is the Epact. If it be
just 30, then is the Prime and the Epact both
one; if short of 30, that Number whatever
it is, is the Epact. Note that the Prime
changes 1. January, and the Epact 1. March.

P R O P. 3.

By the Epact, To know the Age of the Moon.

Add to the day of the Month the Epact,
and so many days more as are Months
from March to the Month you are in, in-
cluding both Months; and if they come not
to 30, so much the Moons Age: If they pass
30, take away 30, and the Remainder is
the Moons Age. This is when the Month
hath 31 days, but if it have only 30 days,
deduct 29, &c. from the excess above 29,
by the same reason.

P R O P.

P R O P. 4.

By the Age of the Moon, to find the time of her coming to the South.

Multiply the Moons Age by 12, and divide the Product by 15, the Quotient will shew the hour of the Moons coming to the South; and if any thing remain, multiply that by 4, and it will shew the Minutes to be added to the hours of the Quotient, and so you shall have the time of the Moons coming to the South : Or by the Table following you may readily find the time of the Moons coming to the South.

T 5

Days

Days H. M. Hours H. M.

The Days of the Moons Age.	1	0.45	The Hours of the Moons age.	1	2
	2	1.38		2	4
	3	2.26		3	6
	4	3.15		4	8
	5	4. 3		5	10
	6	4.53		6	12
	7	5.41		7	14
	8	6.30		8	16
	9	7.19		9	18
	10	8. 8		10	20
	11	8.56		11	22
	12	9.45		12	24
	13	10.34		13	26
	14	11.23		14	28
	15	12.11		15	30
	16			16	32
After noon.		17	34		
		18	36		
		19	38		
		20	40		
		21	43		
		22	45		
		23	47		
		24	49		
		</			

The use of the Table according to the Age of the Moon. Take the Hours and Minutes opposite thereunto in the Table; adding for every hour 2 Minutes more for her mean motion; and the Total shall shew the hour of the Moons coming to the South.

For Example.

Suppose the Moons Age be any day at Noon 10 days and 8 hours old. Look in the Table, and against 10 days you find ——— 8 h 8 m. for the 8 odd hours — 16 m. To which 8 Hours 24 m. allowing 2 Minutes for each Hour for her mean motion is — 17 m. All which added together shews she comes to the South at ——— 8 h. 41 m.

To know at what hour the Moon comes to the South, by the following Table.

Days after the Change or Full the Moon is South at,

0	1.	2.	3.	4.	5.	6.	7.
12	12.48	1.36	2.24	3.12	4.0	4.48	5.36

Days after either of the Quarters the Moon is South at.

15	6.48	7.36	8.24	9.12	10.0	10.48	11.36
----	------	------	------	------	------	-------	-------

To know the Hour of the Night by the Moons shadow upon a Sun-Dyal.

☾	H.	M.	☾
1	0	0	29
2	10	24	28
3	9	36	27
4	8	48	26
5	8	0	25
6	7	12	24
7	6	24	23
8	5	36	22
9	4	48	21
10	4	0	20
11	3	12	19
12	2	24	18
13	1	36	17
14	0	48	16
15	0	0	15

First know the Age of the Moon, and if it increase, from the shadow subtract these Hours and Minutes: If it decrease, add the same to the shadow, and it will give you the true hour.

P R O P. 5.

By knowing the time of the Moons coming to the South, to know the time of the Night by the shadow of the Moon upon a Sun-Dyal.

Observe by a Sun-Dyal as if you would see what a Clock it were by the Moon, and observe how much the shadow of the Moon doth either want, or is past the hour of 12 upon the Dyal: For so much it doth want of, or is past the time of the Moons coming to the South. *As for Example.*

Suppose as before the Moon did come to the South at 8 hours, 41 minutes after noon or at night, and the shadow of the Moon upon the Dyal were at 10. This wants 2 hours of 12, and therefore it wants 2 h. of 8 h. 41 m. so that it is six of the Clock, and 41 m. But if the shadow had been at 2 upon the Dyal, then you must have added 2 h. to the Moons coming to the South, and so it had been 10 h. 41 m. at night: The like is to be done by any other hour whatsoever.

S E C T. 5.

To know the hour of the Night by the Stars.

1. To do this by the Globe, Rectifie the Globe to the Latitude of the place, and bring the Suns place to the Brazen Meridian, and the Index of the hour-circle to 12: Then bring any Star to the same Altitude on the Globe that it hath in Heaven, and
the

the Index of the hour-circle will point to the hour of the Night.

2. *By Mr. Gunter's Nocturnal.*

This Nocturnal is a very easy and ready way to find the hour of the Night by the Stars, wherein the Center upon which it moves is supposed the North-Pole, having several Constellations near the same Pole described upon the Nocturnal, with a Circle of Months fixed lying without the Rundle, as also an hour-Circle of twice 12 h. one of the 12 signifying 12 at Noon, the other 12 at Midnight; and the line supposed to be drawn betwixt 12 and 12, thorough the Center is the Meridian: Now for the use thereof, first look up to Heaven to the North Pole, and observe what Stars are upon or near the Meridian in Heaven: Then place the same Star to the like situation upon the Rundle, and against the day of the Month you will find the hour of the Night.

3. *In the third place, To find the hour of the Night by the Stars coming to the Meridian; their Rising, and Setting, and Right Ascension.*

How to find the Right Ascension of any Star by the Globe is taught, pag. 363, and is no more but instead of bringing the Suns place to the Meridian, you must bring the Star, (whose Right Ascension you seek) to the

the Meridian, and the number of degrees contained betwixt the vernal colure and that degree of the Æquinoctial which comes to the Meridian with the Star, is that Stars right Ascension: Also by the Globe to find the time of any Stars Rising or Setting, or coming to the Meridian, is taught *pag.* 364. How to find the Right Ascension of any Star by Theakers Planisphere is shown *pag.* 374: As also their rising and setting, and their coming to the Meridian, is shown *pag.* 375.

To find the Suns Right Ascension is the same both by the Globe and Planisphere, by bringing the Suns place to the Meridian instead of bringing the Star to the brazen Meridian on the Globe, or the paper Meridian of the Planisphere, as is explained in the fore-mentioned places, *pag.* 362, 374, you have also the Analogy or Proportion to work it by Arithmetically, *pag.* 350. So I need not here repeat any of them.

But shall shew you how by the Right Ascension of the Sun and Stars to find Arithmetically the time of the Stars coming to the South, and thereby the Stars hour, and by all these the hour of the night.

1. *To find at what hour any Star will be upon the Meridian.*

Add the Complement of the Suns Right Ascension for the day proposed to the Right Ascension of the Star, the Sum of them together will be the time of that Stars being upon the Meridian.

For

For Example.

h. m.

The Bulls Eye 1st January Right Ascension. ——— 4 17

Compl. of the Suns Right Ascension is ——— 4 25

The Bulls Eye upon the Meridian. ——— 8 42

That is, 42 Minutes past 8 at Night.

It is noted by *Norwood* that any star in 14 days comes to the Meridian just an hour later, and in a Month two hours.

2. *To find the Rising and Setting of the Stars.*

The Semidiurnal Arch of any Star, which is half the time the Star appears above the Horizon, subtracted from the time of the Stars being upon the Meridian, giveth the time of the Stars Rising, and the same being added thereunto, giveth the time of the Stars setting; and knowing the time of the Stars Rising or Setting, and coming to the Meridian; and seeing any of the said Stars rise, set, or come to the Meridian in Heaven, hereby you know the **Exact Hour** of the Night.

3. *But to find the Hour of the Night at any other time, you must know the Stars hour.*

Now the Stars hour is the horary distance, or the number of hours that the said Star is distant from the Meridian, or so many Hours as the Star is short of, or past the Meridian.

Now

Now to find the Hour of the Night at any time by any of the Stars.

Add the Stars Hour, the Stars Right Ascension, and the Complement of the Suns Right Ascension, all three together, the Sum is the hour of the Night. As for Example.

The Bulls Eye 11th of December.

The Stars Hour is	— — — — —	8	56
The Stars Right Ascension is	— — — — —	4	17
The Complement of the Suns Right Ascen.	6	9	
		Sum	19 22

Now because this exceeds 12, you must deduct 12 from it ; so that the true hour of the Night is 7 h. 22 Minutes : And note further, that when this Sum is short of 12, it shews the hour at Night ; when it exceeds 12, it shews the hours next Morning: So that in this Example it is 7 h. 22 m. in the Morning ; and herewith I shall conclude.

Having finished what I intend to say concerning *Astronomy*, and the Resolution of these Astronomical Questions of the Sphere, being the third and last Mathematical Liberal Science according to my Division of them in the beginning : I shall therefore here set up a *Non Ultra* at this time. Having (I hope) in some measure shewed you how to Measure the *Earth*, the *Sea*, and the *Heavens*, principally and especially by *Trigonometry*. Therefore in allusion to a *Triangle*, the most excellent of Geometrical Figures, and the other *Ternaries* mentioned here, and elsewhere, pag. 2, 3. I say in allusion to these, To the Tri-un Deity, The Trinity in Unity, and Unity in Trinity, who created all these Things of Nothing in Number, Weight and Measure, by his Infinite Wisdom and Power, and doth Uphold and Govern All by his Providence, and design'd all for his Glory, to him be all the Glory and Honour, and Adoration in *Secula Seculorum*. A M E N.

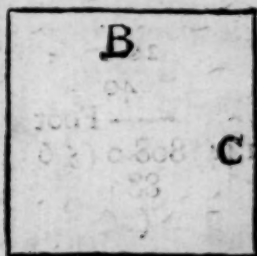
T H E

THE
Theory and Practice
OF
MEASURING.
Both by the *Pen* and *Gunter's-Line*.

Of Surfaces, or Superficial Measure.

A SQUARE.

LET the Side of a *Square* be B
or C, = 7 Inches each; mul-
tiply B by C, and divide by 144,
and the *Quotient* is 34 Feet.



Vulgarly.

Example.

B=70

C=70

Or multiply 5
Foot 10 Inches (for 70 Inches is 5
Foot 10 Inches) by 5 Foot 10 Inches,
cross ways, by saying 5 times 10
Inches is 4 Foot 2 Inches, and 5
times 5 is 25, which makes 29; then take the $\frac{1}{2}$
and the $\frac{1}{4}$ of the first Line, as follows, for the 10
Inches in the last Line, which was not multiply'd,
as follows :

Practically.

Of Measuring.

Practically.

5 10 Inch.

5 10 Inch.

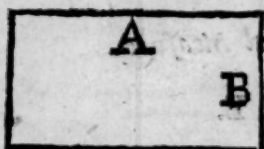
29 2

2 11 = $\frac{1}{2}$ 1 11 = $\frac{1}{3}$

34 0

By *Gunter's* Line and Compasses, set one Point in 1, and extend the other to 5 and $\frac{1}{2}$, on the Line of Numbers; then turn the Compasses, the Point standing still in 5 $\frac{1}{2}$, and the Point that stood in 1, will cut the Rule 34 Foot. Do this for *Squares* only.

An OBLONG.



Let the Side A be 40, B 20.2, A multiplied by B, and divided by 144, gives 5 Foot 7 Inches.

Example.

Foot Inch.

3 4
1 83 4
1 8 = $\frac{1}{2}$ 7 = $\frac{1}{3}$ of that $\frac{1}{3}$

5 7

20.2
40
— Foot
144) 808.0 (5.6
880
(16

By *Gunter*, set one Foot in 1, and extend the other Foot to 1 Foot 8 Inches, or a little less than $\frac{3}{4}$, and then set the Point in 3 Foot $\frac{1}{3}$, and the other Point will cut 5 $\frac{1}{3}$ Inches.

A TRIANGLE.

All *Triangles* are measur'd by one and the same Method, by letting fall a Line from the bluntest *Angle* to the nearest Place of the opposite Side, and that is called the *Perpendicular*. The upright Side of a Right *Angle Triangle*, is the perpendicular.



Let the Side A.B be 42 Inches, and the *Perpendicular* C.D 66 : Multiply A.B by $\frac{1}{2}$ C.D, and the *Product* divide by 144.

Example.

Vulgarly.

42 = A.B.

33 = $\frac{1}{2}$ C.D.

Practically.

3 6 = A.B.

2 9 = $\frac{1}{2}$ C.D.

Decimally.

3.50

2.75

126

126

7 0

1 9 = $\frac{1}{2}$ A.B.

10 $\frac{1}{2}$ = $\frac{1}{2}$ A.B.

1750

2450

700

144) 1386 (9.6

90.0

3 6

9 7 $\frac{1}{2}$

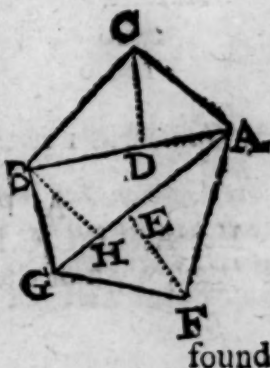
9.625

Note, When you work Decimally, 1.5 is = $\frac{1}{2}$ a Foot .25 to a Quarter, and .75 to 3 Quarters.

By *Gunter*, as 1. is to 3. $\frac{1}{2}$, so is 2. $\frac{3}{4}$, to 9 Foot 7 $\frac{1}{2}$ Inches.

A TRAPEZIUM.

This Trapezium is divided into three *Triangles* by the Lines A.B. and A.G. and the *Perpendiculars* are C.D. B.H, and E.F. The several Contents of those *Triangles* being found in Inches, according to the last Paragraph, and added together, divide by 144, and it gives the Content of the *Trapezium* ; or if they be



found in Feet and Inches, add them together, and the Sum is the Content.

Suppose the *Content* in Inches of the *Triangle* A.B.C. to be 800 Inches; the *Triangle* A.B.G. 634, and A.G.F. 824 Inches, the Sum is 225; which divide by 144.

Example.

$$\begin{array}{r} 144) 2258 \text{ (15.6)} \\ 818 \\ 98.0 \\ 126 \end{array}$$

By *Gunter*, as 1 is to the $\frac{1}{2}$ of C.D, so is A.B. to the Content of the *Triangle* A.B.C; and so work the other two, and make a Sum, and you have the Content of the *Trapezium*.

A RHOMBOIDES.



Let A.B. be 640 Inches, or 53 Foot 4 Inches; A.C. 200 Inches, or 16 Foot 8 Inches. Multiply A.B. by A.C, and divide by 144.

Example.

$$\begin{array}{r} 640 \\ 200 \\ \hline 144) 128000 \text{ (888.8)} \\ 1280 \\ \hline 1280 \\ 128.0 \end{array}$$

$$\begin{array}{r} \text{Feet. Inch.} \\ 53 \text{ 4} = \text{A.B.} \\ 16 \text{ 8} = \text{A.C.} \end{array}$$

$$\begin{array}{r} 853 \text{ 4} \\ 26 \text{ 8} = \frac{1}{2} \text{ of A.B.} \\ 8 \text{ 10} = \frac{1}{2} \text{ of the } \frac{1}{2} \text{ A.B.} \end{array}$$

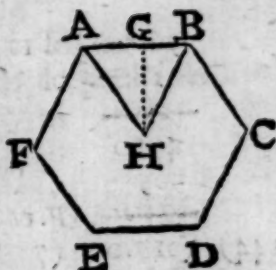
$$\hline 888 \text{ 10}$$

By *Gunter*, as 1 is to A.B, so is A.C. unto the Content 888 $\frac{1}{2}$.

A POLYGON.

Let A.B, B.C, C.D, E.F, F.A, be the Sides of the *Polygon*, each of them equal to 30 Inches, and perpendicular.

pendicular G.H. (being a Line drawn from the Center to the middle of any one of the Sides) equal to 20. Multiply the Sum of all the Sides by $\frac{1}{2}$ of the Perpendicular G.H, and divide by 144; or multiply the Content of the Triangle A.B.H. by 6, the Number of the Sides, and you have the Content of the Hexagon. It needs no Example, because it is wrought as Triangles.



Note, To find the Center of a Polygon of odd Number of Sides, draw a Line from any Angle to the Middle of the opposite Side, and cross that with another, and where they cross is the Center. But if of even Sides, draw a Line from any Angle to the Opposite, and cross that.

By Gunter, as 1 is to $\frac{1}{2}$ G.H, the Perpendicular from the Center to the Middle of one Side, so is the $\frac{1}{2}$ Sum of all the Sides to the Content.

A CIRCLE.

A.B. the Diameter, A.B.C.D. the Circumference, E. F. the Chord, E.H F. G. the Sector, E.C.F. the Segment, F.G, or B. G. Cemi Diam. F.H a Sine, C.I. ver. Sine.



Before you can find the Content of a Circle, you must have both Diameter and Circumference given.

Let the Diameter A.B. be 36 Inches; multiply 36 by 3.14, and the Product will be 113 Inches = the Circumference; multiply $\frac{1}{2}$ the Circumference by $\frac{1}{2}$ the Diameter, and the Product divide by 144, gives the Content in Feet.

Example.

Example.

56.5 = $\frac{1}{2}$ of 113 A.B.C.D. 4 8 = $\frac{1}{2}$ the Circumf.
 18 = $\frac{1}{2}$ of 36 A.B. 1 6 = $\frac{1}{2}$ the Diamet.

4520
 565

Feet Inch.

4 8

2 4 = $\frac{1}{2}$ the first Line

7 0

244) 1017.0 (7 0
 90

By Gunter, as 1 is to $\frac{1}{2}$ the Circumference, so is $\frac{1}{2}$ the Diameter to the Content.

A Sector of a CIRCLE.

Multiply E.G. by E.C. and the Product is the Content.

A Segment of a CIRCLE.

Take the Content of the Triangle E.F.G from the Content of the Sector E.C.F.G, and the Remainder is the Content of the Segment E.C.F.

Or suppose the Segment alone be required, and you have no Content of the Sector, then draw the Line E.H. to the middle of F.G. Subtract H.F. from E.C.F, and multiply by the $\frac{1}{2}$ of G.F, and you have the Content.

Note, That the Cord E.F. is always .9 of the Arc E.G.F.

An O V A L.

Let A.B be 48 Inches, and C.D 72 ; multiply C.D by A.B, and extract the Square Root of the Product, and that is the mean Diameter of the Oval.



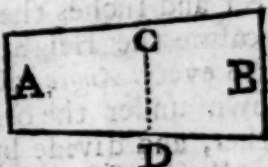
The Root of 72 multiply'd by 48, is 58 ; multiply that by 3.14, and that gives 182 = the Circumference of a Circle, whose Diameter is 58 ; multiply $\frac{1}{2}$ the Circumference by $\frac{1}{2}$ the Diameter, as you did in a Circle, (for now it's reduc'd into a Circle) and divide by 144, or 282, or 231, as before, and you have the

Content in Superficial Feet, or Area in Gallons.

Suppose

Suppose A.B a Plank or Board, broader at one End than the other.

Let the End A be 39 Inches, or 3 Foot 3 Inches, and the End B, 30 Inches, or 2 Foot 6, Length, 120 Inches. Multiply A by B, & the Square Root of the Product is the mean Breadth at C.D, which is the true Geometrical Mean; but the Practical way is, to add both Ends together, and take the $\frac{1}{2}$ of the Sum for a Mean. Multiply that Mean into the Length, and divide by 144 for the Content.



Example.

39	34.2	Feet. Inch.
30	12.0	10 00
<u>Mean</u>	<u>6840</u>	<u>02 10</u>
1170 (34.2	342	20 00
270	1026	05 00
64) 14.00	<u>Content</u>	<u>03 40</u>
682) 36	4104.0 (28.50	28 04
	1224	<u>Content.</u>
	720	
	66	

By Gunter, as 1 is to the Mean C.D, 2 Foot 10 Inches, so is the Length 10 Feet to 28 Feet.

Let a Plank be ever so irregular, it may be measured pretty exactly, by taking the Breadth of it in 5 or 6 Places, and add the several Breadths together, and that Sum divide by the Number of Places you have taken the Breadths, (suppose 6) and the Quotient is the mean Breadth.

Instructions to Painters and Glaziers.

As Rooms are of so many various Forms, I shall only give a general Rule in all Cases.

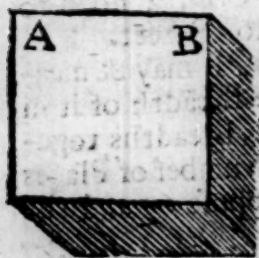
Take a String, and apply one End of it to any one Corner of the Room, then measure the Room, going into

into every Corner, with the String, until you come at the Place you began at; then measure how many Feet and Inches the String is, and set down; then measure the Height by the same Method, going into every Angle of the Molding, and set that down under the other; multiply the one by the other, and divide by 9, and the Quotient will be the Yards of Painting, or Wainscot. If there be a Chimney, measure the Chimney by the foregoing Rules, according to the Figure of it, and deduct from the whole.

Glaziers are to take the Depth and Breadth, of all Windows that are alike, in Inches, and make a Sum of all the Depths, and likewise of all the Breadths, and multiply the one by the other, and divide by 144, and the Quotient is Feet; or by the Rule, as 1 is to the Height or Depth of the Whole, so is the Breadth of the Whole to the Content.

Measuring of Solids, apply'd to Timber, and Liquids, such as Beer and Wine-Measure, &c.

A C U B E.



A Cube is a Solid whose Length, Breadth and Thickness are equal, in the Form of a Dye.

Let A.B be 39 Inches; multiply A.B by A.B; that is, 39 by 39, and that Product by 39, and divide by 1728 for Timber, 282 for Beer, 231 for Wine, and the several Quotients gives you the Feet of Timber, or, Gallons of Beer or Wine that Cube would hold if it were hollow.

Example

Example.

Vulg.	Pract.	Decim.
39	3 3=A.B	3.25
39	3 3=A.B	3.25
351	9 9	1625
117	10 $\frac{1}{2}$	650
1521	10 6 Sum	975
39	3 3=AB	10.5625
13689	31 6	3.25
4563	2 $7\frac{1}{2}$	328125
1728) 59319 (34.3	34 $1\frac{1}{2}$ Sum of	211250
7479		316875
567.0		
486		34.328125

By *Gunter*, set one Foot of the Compasses in 1, and extend the other to the Length of the Side in Feet and Parts, viz. 3 Foot and $\frac{1}{4}$; then turn your Compasses twice at that Extent, and the Point will cut 34 Foot on the Rule, and somewhat more.

A PARALLELEPIPEDON.

Let A.B the Length be 144 Inches, or 12 Foot; and B.C 8 Inches, or one Foot and a half; and Thickness or Breadth of one End C.D, 15 Inches, or Foot 3 Inches.



Multiply B.C, the longest Side of the End, by C.D. the shortest, and multiply the *Product* by the Length A.B, and divide by 1728, and the Quotient is the Content in Feet.

Example.

BCxCDxAB

 1728=22 5
 282 for Beer.
 231 for Wine.

18
 15

 90
 18

 270
 144

 1080
 1080
 270

 Feet
 1728) 38880(22.5
 4320
 874.0
 110

Feet. Inch.
 1 6= B.C
 1 3= C.D

 1 6
 a 4 $\frac{1}{2}$ = $\frac{1}{4}$ BC

 1 20 $\frac{1}{2}$
 12 00=AB

 12 00
 6 00= $\frac{1}{2}$ AB
 4 00= $\frac{1}{4}$ AB

 22 00 $\frac{1}{4}$

By *Gunter*, all Solids that are longer than broad, must be done at two Operations; as 1 is to 1 Foot 6 Inches, or a half, so is 1 $\frac{1}{2}$, or 1 Foot 3 Inches to 1 Foot 10 $\frac{1}{2}$. Then, as 1 is to 12 the Length, so is 1 Foot 10 $\frac{1}{2}$ to 22 the Content.

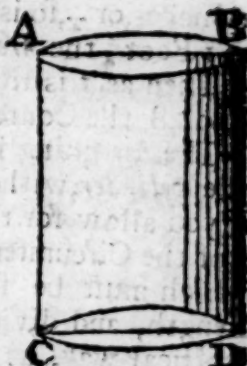
Thus, set 1 Foot in 1, and extend the other to 1 $\frac{1}{2}$, then set one Foot in 1 $\frac{1}{2}$, and the other Foot will reach 10 $\frac{1}{2}$, which is 1 Foot, and above $\frac{1}{2}$. Then alter your Compasses, by setting 1 Foot in 1, and extend the other to 12 on the Rule, then at that Extent, set one Foot of the Compasses in 1 Foot 10 Inches $\frac{1}{2}$, and the other will extend to 22,3.

Cont

A 172

A CYLINDER.

A Cylinder is a Body of equal Diameters in all Parts, and may be either Solid, like the Rowling-stone of a Garden, or hollow like a Tub, for holding Liquor.



Let AB the Diameter be 40 Inches, or 3 Foot 4 Inches, and BD 66 Inches, or 5 Foot 6 Inches. Before you can find the Area of the End AB, you must find the Circumference, by multiplying the Diameter into 3.14. Then half the Diameter, viz. 20, being multiply'd into $\frac{1}{2}$ the Circumference 62.8, produces 125.6 square Inches, the Area of one End; multiply that Area by the Length B.C 66 Inches, and you have the Content of the Cylinder in solid Inches; Divide by 1728 for Feet of Timber, or 282, or 231 for Gallons of Beer, or Wine.

Example.

$$\begin{array}{r} A.B = 3.14 \\ \quad = 40 \end{array}$$

$$\begin{array}{r} \text{Circumf. } 125.60 \\ \text{Circumf. } 62.8 \\ \text{Diam. } 20 \end{array}$$

$$\begin{array}{r} \text{Area } 125.6 \\ \quad 66 \end{array}$$

$$\begin{array}{r} 7536 \\ 7536 \end{array}$$

$$\text{Content } \text{---}$$

$$\begin{array}{r} 1728 \quad 82896 \quad (47.9 \text{ Feet}) \\ 13736 \\ 1680.0 \\ 1158 \end{array}$$

$$\begin{array}{r} \text{Feet. Inch.} \\ 5 \text{ } 3 \frac{1}{2} \text{ Circumf.} \\ 1 \text{ } 8 \frac{1}{2} \text{ Diam.} \end{array}$$

$$\begin{array}{r} 4 \text{ } 5 \text{ } 1 \text{ } 2 \\ 2 \text{ } 7 \frac{1}{2} \text{ of the } \frac{1}{2} \text{ Circumf.} \\ 10 \frac{1}{2} \text{ of the } \frac{1}{2} \text{ Cir.} \end{array}$$

$$\begin{array}{r} 8 \text{ } 9 \\ 5 \text{ } 6 \text{ } = \text{BD the Length.} \end{array}$$

$$\begin{array}{r} 43 \text{ } 9 \\ 4 \text{ } 5 \frac{1}{2} \\ 48 \text{ } 2 \frac{1}{2} \text{ Feet} \end{array}$$

By Gunter, as 1 is to $\frac{1}{2}$ the Circumference 5 Foot 11 Inches, or $\frac{1}{4}$, so is $\frac{1}{2}$ the Diameter, 1 Foot 8 Inches to 1 Foot $\frac{1}{2}$ the Area of the End A.B.

Then as 1 is to the Length 5 $\frac{1}{2}$, so is the Area 8 $\frac{1}{2}$ to 48 the Content in Feet.

The foregoing Rule gives the Solid Capacity of the Cylinder, without any Deduction for Slabs; but if you allow for the Slabs, then you are to take the $\frac{1}{4}$ of the Circumference for the Side of the Square, which must be squar'd and multiply'd into the Length, and divided by 1728, or multiply in the Practical way, as you did in the Parallelepipedon.

A C O N E.

A Cone is the $\frac{1}{3}$ of a Cylinder of the same Height, and Diameter equal to the Base of the Cone.

Before you can measure a Cone you must find the Perpendicular Height C.D, by subtracting the Square of the $\frac{1}{2}$ Base B.D, from the Square of the slant Height C.B, and the Remainder is the Square of C.D the Perpendicular; extract the Square Root, and you have the Perpendicular C.D.



Let A.B be 40 Inches, or 3 Foot 4 Inches; C.D 66 Inches, or 5 Foot 6 Inches. Find the Area of the Base, as you did in a Cylinder, and multiply by $\frac{1}{3}$, the Height C.D, and divide by 1728 for Timber, 282 for Beer, 231 for Wine.

Example.

Example.

1 1/2 Circumf. 62.81 2 1/2 Foot Inch.
 1 1/2 Diam. 20 5 3/4 Circumf.
 1 1/2 1256 1 8 1/4 Diameter
 C.D = 66
 Prod. 82896 Content
 1728) 1/3 Prod. 27532 (15-9
 10352
 1712.0
 1568
 8 9
 4 4 1/2
 2 11
 16 0 1/2 Content.

By *Gunter*, as 1 is to the $\frac{1}{2}$ A.B, so is the $\frac{1}{2}$ A.B.D the Circumference, to the *Area* of the Base. Then as 1 is to $\frac{1}{2}$ the Altitude C.D, so is the *Area* to the Content.

A Square Pyramid, or any other, let the Form of the Base be what it will, you must always multiply the Area of the Base by $\frac{1}{3}$ the Altitude, and divide as before. The Superficial Area of a Cone is found by multiplying the Circumference of the Base by $\frac{1}{2}$ the slant Height.

The Frustum of a C O N E.

E.F.A B in the foregoing Figure, is the *Frustrum*,
or piece of a *Cone*.

Let E.F be 24 Inches, and A.B 36; add $\frac{1}{2}$ the Difference of the two Diameters to the lesser Diameter, and that is the mean Diameter; multiply that by 3.14, and that gives a mean Circle to that Diameter; multiply half of that Circle by half the Diameter, and the *Product* multiply by the Depth G E, and divide by 1728, or 282, or 231, and you have the several Contents.

A GLOBE or SPHERE.



The Superficial Area of a Sphere, is equal to four times the Area of its greatest Circumference.

If a Circle be describ'd, whose Diameter is equal to twice the Diameter of the Sphere, the Area of that Circle is equal to the Sur-

face of the Sphere.

Multiply the Circumference of the Sphere by the Diameter, and multiply the Product by $\frac{1}{6}$ the Diameter, and divide by 1728 for Timber, 282 for Beer, 231 for Wine,

Example.

3.14	2826.9 Product.
A.B=30 Diam.	9= $\frac{1}{6}$ the Diam.
94.23 Circumf.	1728) 14134. (8 Foot.
30 Diam.	(310

2826.9 Product.

To find the Content of the Segment c.d.e.f, multiply the Altitude e.f by 3, and the Product by the Square of e.f, and to the Product add the Cube of e.f, and multiply the Sum by .5235.

Example.

Radius e.f=6	351 Sum
6	.5235 Multipli.
36	1755
Tripl. Alt. =9	1053
	702
324	1755
27 Cube of the Alt.	183.7485 Content.
351 Sum.	To

To Measure Timber or Stone.

Suppose A.B.C a Piece of Timber 28 Foot Long, and the Circumference at A.B, 24 Inches, at E.F 20 Inches, at G.H 16 Inches; add those three together, and the $\frac{1}{3}$ of the Sum is accounted the Mean Circumference, the $\frac{1}{4}$ of which, call the Mean Side of a Square: Multiply that by its self, and multiply the Product by the Length, and divide by 1728, and the Quotient is the Content.



Example.

A.B=24

E.F.=20

G.H=16

— Inch. Inch.

3) 60 (20 = 5

5=Side of a Square.

5

25

336 Length in Inches.

1728) 8400 (4.8

1288.0

1056

Four Foot and EighthTenths is the Solid Capacity, considering it as a Square Body, by cutting off the *Slabs*, for those who buy *Timber*, measure it according to the Square within, which they commonly deem to be $\frac{1}{4}$ the Girt. But if you design to have the true Content of it, the *Slabs* not being deducted, then you must measure it as the *Cylinder*, making the Mean Diameter, the Diameter of the *Cylinder*.

All irregular Bodies must be reduc'd, as near as possible, into regular Figures, according to the Judgment of the Artist; so that there can be no particular

1st Rule set down, by reason the Figures of Bodies are so various: But he who has well digested the foregoing Rules, may easily form a Method to take the Dimensions of any irregular Body, so that the Error shall not be great.

To Measure a Spheroid, or Solid Oval like an Egg.

Find the Mean Diameter, by extraction of the Square Root, as directed in the *Oval*, then by that Diameter find the Circumference, and the $\frac{1}{2}$ Circumference, being multiplyed into $\frac{1}{2}$ the Diameter; multiply the Product by $\frac{3}{2}$, the longest Diameter, and you have the Solid Content.

To Measure the middle Frustum of a Spheroid, such as a Wine-Pipe.

To the double Square of the Bung Diameter, add the Square of the Head; multiply the Sum by the Casks Length, and divide by 1077, and the Quotient is the Number of Gallons of Beer the Cask will hold.

How to Measure most sorts of Artificers Work, such as Masons, Bricklayers, Carpenters, Joiners, Plasterers, Glaziers, Painters and Paviers.

Artificers have different ways of Measuring, according to the Custom of each Trade; some give the Content in Rods, others in Yards; some in Feet, others in Squares, and some in Running-Measure, that is, only Length without any Breadth, by a Line.

The only practical way is, after we have taken all the Dimensions in Feet and Inches, we multiply so many Feet and Inches, by so many Feet and Inches, and that produces the Content in Feet and Inches, which we may reduce afterwards into Square Rods, Yards, &c. according to Pleasure.

Let the Dimensions be 24 Foot 10 Inches, by 8 Foot 9 Inches.

Example.

Example.

Practically.

Decimally.

$$\begin{array}{r}
 24 \text{ } 10 \\
 8 \text{ } 9 \\
 \hline
 198 \text{ } 8 \\
 12 \text{ } 5 = \frac{1}{2} \text{ of } 24 \text{ } 10 \\
 6 \text{ } 2\frac{1}{2} = \frac{1}{4} \text{ of } 24 \text{ } 10 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 12) 80 \text{ (6 Foot } 24 \text{ } 76 \\
 \text{ (8 Inches) } 8 \text{ } 75 \\
 \hline
 12380 \\
 17332 \\
 \hline
 19828
 \end{array}$$

217 $3\frac{1}{2}$ Content.

216.65.00

The manner of working the Vulgar Way is this. Multiply 24 Foot 10 Inches, by 8 Foot 9 Inches cross ways; thus 8 times 10 Inches is 80 Inches, divided by 12 is 6 Foot 8 Inches: Set down the 8 Inches, and carry the 6 Foot in Mind; then say, 8 times 4 is 32, and 6 I carry'd is 38: Set down the 8 under Feet, and carry 3, and 8 times 2 is 16, and 3 I carry'd is 19, which set down before the 8, and that makes 198 Foot 8 Inches: Now you have multiply'd the first Line into the Number of Feet in the second Line, then as 9 Inches is the $\frac{3}{4}$ of a foot, you must take the $\frac{3}{4}$ and the $\frac{1}{4}$ of the first Line, and set them down in their proper Places, under what was set down before: or it is all one, to take the $\frac{3}{4}$ of the first Line for the $\frac{1}{2}$ Foot, and take the $\frac{1}{4}$ of that $\frac{1}{2}$ for the quarter Foot; make a Sum, and you have the Content 217 $3\frac{1}{2}$. You must always consider what *aliquot* Part or Parts, the Inches in the last Line is; take that *aliquot* Part of the first Line, and set down in order, Inches under Inches, and Feet under Feet, and let this be your general Rule in all Cases.

Aliquot Parts of a Foot are as follows.

Inches.	Inches.
1 is $\frac{1}{12}$	7 is $\frac{1}{2}$ and $\frac{1}{12}$ or $\frac{1}{3}$ and $\frac{1}{4}$
2 is $\frac{1}{6}$	8 is $\frac{2}{3}$ and $\frac{1}{6}$ or $\frac{3}{4}$
3 is $\frac{1}{4}$	9 is $\frac{3}{4}$
4 is $\frac{1}{3}$	10 is $\frac{5}{6}$ and $\frac{1}{6}$
5 is $\frac{5}{12}$ and $\frac{1}{12}$	11 is $\frac{11}{12}$ and $\frac{1}{12}$
6 is $\frac{1}{2}$	

To

To Measure Bricklayers Work. (Alingham.)

Brickwork is commonly measur'd by the Rod-Square, a Rod being 16 Foot and a $\frac{1}{2}$, and a Rod-Square is produced by multiplying that Rod by it self, which produces 272 $\frac{1}{2}$ Foot, the Content of a Rod-Square

If a Wall be over or under a Brick and a $\frac{1}{2}$ thick, it must be reduced into that Thickness, by reducing all the Work that is measured, into $\frac{1}{2}$ a Brick thick, and then turning it all into a Brick and $\frac{1}{2}$, by the following Rule.

Suppose one Dimension to 56 Foot, by 24 Foot and 6 Inches, one Brick thick, another 18.6 by 6.9, two Bricks thick, and another 22.4 by 8.6, three Bricks thick. Set down the Dimensions as follows, and multiply the Total of 1 Brick by 2, the Total of 2 Bricks by 4, and of 3 Bricks by 6, and the Sum of all is the Total Square, Feet at $\frac{1}{2}$ a Brick thick, which divide by 3, and it gives the Content in Feet 1 Brick thick, divide by 272, and it gives the Rods.

Example.

56 0 = 1 B. thick	18 6 = 2 B.th.	22 4 = 3 B.th
24 6	6 9	8 6
1344 0	111 0	178 8
28 0	9 3	11 2
1372 0 Foot	4 7 $\frac{1}{2}$	189 10 Foot
2	124 10 $\frac{1}{2}$ Foot	6
2744	4	1139
499		
1139	499 6	

4382 the Sum of all the Totals.

3) 4382 (1460 Feet, 1 B. $\frac{1}{2}$ thick.

13
18
02

272) 1460 (5.3 Rods 1 B. $\frac{1}{2}$ thick.

100.0
184

In

In measuring a *Chimney*, if it stand alone without leaning against any thing, girt it about below the *Mantle*, and take that for the *Length*, and the *Height* of the *Room* for the *Breadth*, and multiply them together, gives the *Content*! But if it stand against a *Wall*, then you must measure it round to the *Wall*, and multiply that into the *Height* of the *Room*, and that's the *Content*. Allow nothing for the *Vacancy* between the *Hearth* and *Mantle-Tree*.

For *Arches*, take the *Breadth* and half *Breadth* of the *Arch*, and add them together, multiply the *Sum* by the *Length*, for the *Content* in the *Thickness* of the *Arch*.

All kind of *Ornamental Work*, such as *Cornice*, or *Frieze*, &c. are measur'd by *Running-Measure*.

Carpenter's Work, as *Flooring*, *Roofing*, &c. are measured by the *Square*, which contains 100 *Square-Feet*.

Mason's Work is measur'd by the *Foot*; if you measure the *Workmanship* of *Stone-steps*, then you must measure both *Ends* of the *Steps*, as well as the *Length*.

N. B. The method of *Measuring Timber*, by taking the mean *Thickness* by the *Callopers*, is very erroneous; for they make the *Extent* of the *Callopers* the side of the *Square*; which being square, and multiply'd into the *Length*, will make its *Content* $\frac{2}{5} \frac{1}{6} \frac{1}{7}$ or $\frac{1}{8}$ (according to its *Irregular form*) more than it truly is; and yet this is the *Method* of the *Officers* on the *River Thames*.

How to Gauge all manner of *Brewing Vessels* without the *Line* of *Inches*, or *Division*. For the *Square* of the *Diameter* is the *Area* in *Gallons*; and so are the *Sides* of *Squares* multiplied together without any more working.

THE Law hath order'd, that a *Beer-Gallon* shall contain 282 *Cubical Inches*, & a *Wine-Gallon* 231; and those two *Numbers* all *Gaugers* make use of for *Divisors*; for they multiply the one side of a *Square*

Square Cooler, or oblong into the other, according to the true Rules of Mensuration and divide by either of those Numbers, having Regard to the Quality of the Liquor, and the Quotient gives the Area in Gallons upon an Inch deep.

Now to save the Trouble of a long Multiplication and Division, which sometimes occasions Mistakes; I have found out a way to Gauge any Vessel in the tenth part of the time, without either of those Numbers, or Inches. But by multiplying the Sides of Squares, or the Diameter of Rounds, &c. into themselves, and the Product is the Number of Gallons & parts the Vessel holds upon an Inch in depth; and it is so far from being disputed, that I shall hereafter prove it to be just, by working some Questions both ways, and when compared they will be found equal; so that when the following Demonstration is perused and duly considered, I doubt not but impartial Judges will give their Opinion, that it is not only the quickest, but the most correct way, by reason it is so little liable to Errors, which the other tedious way is very subject to.

Now 282 is the Superficial Area of a Square which holds a Gallon or an Inch in depth, therefore the side of that Square must be equal to the Root of 282, viz. 16.796. so that if any Rule be divided into Roots instead of Inches, and those Roots number'd 1, 2, 3, &c. and subdivided into Decimal and Centesimal Parts, that is surely a proper Instrument to Gauge by to find Areas, and to have no regard to Inches, other ways than to multiply that Area by Inches and Tenths to find the Content; for an Area is only the quantity at an Inch deep; therefore, the Liquors Depth must be multiply'd by Inches.

If the Root of 282 be supposed to be 16.8, that is certainly the side of a Square, which being multiply'd by its self, produces a Surface equal to the Superficial Area of a Gallon, which is nothing else but the Boundary of a Gallon in Length and Breadth

without

without any Thickness; but when this receives Thickness, by being an Inch, 2, 3, or 4 deep, then it is a solid Body, and contains so many Gallons and Parts, as it is Inches and Parts deep: which is so very plain, that it admits of no Demonstration.

Since the Root 16.8 is the side of a Square which produces a Surface, which at an Inch deep contains a Gallon; then the Side of a Square equal to twice that Root, must surely produce a Surface equal to 4 times that quantity: for double Diameters produce quadruple Contents, or Area's. And so if you double the Side of a Square, the Content will be quadruple.

This being granted, it must necessarily follow, that as Roots are the Boundaries of several Quantities, those Quantities may justly be measured by those Boundaries, and the Roots may be term'd 1, 2, 3, 4, &c. And subdivided after the same manner as if they were Inches.

The Gauge Point, or 18.95, is call'd the Diameter of a Circle, which holds a Gallon on an Inch in Depth, or the Diameter of such a Circle, whose Superficial Area is 282. If this be call'd the Gauge Point for Circular Gauging, 16.8 may by the same Rule be call'd the Gauge-Point for Squares, for it performs the same office.

Now if I have divided my Rule into so many Gauge Points as the Rule will permit, instead of Inches; and number'd them, 1, 2, 3, &c. and I take my dimensions therewith, and find the Diameter of a Tub to be 2 of those Divisions or Gauge Points; then since double Diameters gives four times the Content, double Gauge Points must do the same: Consequently if you multiply two by two, it gives four Gallons on an Inch in depth, which is the same as if you had multiply'd 37.9 by its self, and divided by 359, being the square of the Gauge Point.

I invented this Rule when I was concern'd in the
Excise

Excise at Malton in Yorkshire, and found it to be of good use in the Gauging large Floors of Malt; for by this Rule after the Dimensions were taken, I had nothing to do, but Multiply the length into the Breadth, and I had the *Area* at once, without any Division or large Multiplication. After that I Gauged a large Back in Mr. *Crosse's* Brewhouse in St. *Giles's*, Mr. *Everard* then Commissioner being present, who compared my *Area* with Mr. *Shepherd's*, and found it to be just, even to the 1000 part of a Gallon, & perform'd in the 10th part of the time which it would have taken up the common way. The Reason I now Publish it is to be of use to the Officers of Excise, who have large Business and many Maltsters.

We'll suppose the Root of 282 to be 16.8 instead of 16.796. There is placed on the side of the Rule over against 16.8 Number 1. which is one Root, and opposite to 33.6 Number 2, which is two Roots, and opposite to 50.4, if so many Inches on the Rule, Number 3, and each of those Numbers is divided into tenths and hundredth parts.

Suppose a Rule to be so divided, take the Dimensions of any Back or Cooler therewith, as if they were Inches and Tenths, &c.



Let the side B be 33.6, and the side A 50.4. Now the Rule being divided as before directed, the side B will be 2, and the side A will be 3. We will work it

both ways and see the difference, if there be any.

$$\begin{array}{r} 50.4 = A \\ 33.6 = B \end{array}$$

Example.

$$\begin{array}{r} A = 3 \\ B = 2 \end{array}$$

$$\begin{array}{r} 3024 \\ 1512 \\ \hline 1512 \end{array}$$

6 Area

$$282) 1693.44 (6.00 \text{ Area}$$

144

There are 33 Figures in one Operation, and but 3 in the other, which makes the difference in working as 10 to 1.

We shall suppose another to have Decimals by this new

Of Gauging.

447

new Rule. As the side A to be 2.1, equal to 35.3 on the line of Inches. And the side B to be 1.45 equal to 24.4 on the line of Inches: This we shall work both ways for another *Example*.

$$\begin{array}{rcl} 35.3 = A & \text{Example.} & A = 2.1 \\ 24.4 = B & & B = 1.45 \end{array}$$

1412

3.045

1412

706

282) 861.32 (3.05

1532

122

The difference is here but $\frac{5}{100}$ parts of a Gallon which is little more than the quantity of a thimble full.

Let the side A be 564.8 on the Line of Inches, which will be equal to 33.62 by the repeating the length of the Rule. And the side B 32.6 Inches, equal to 20.01 on the new Line.

$$\begin{array}{r} A = 564.8 \\ B = 336 \end{array}$$

$$\begin{array}{r} A = 33.62 \\ B = 20.01 \end{array}$$

33888

3362

16944

6724

16944

672.7362 Area. 1

282) 189772.8 (672.9 Area

2057

832

2688

150

Here you see that one of the largest Backs in London is not 2 tenths, in 672 Gallons less.

Suppose you were to find the Area of a Cylinder, or any other Vessel reduced into a Cylinder. You square the Diameter and divide by 359, and the Quot. is the Area in Beer.

Now by this new Rule (because 18.95 is the Gauge Point, or the Diameter of a Circle whose Area is 282) I make 18.95 Number 2 on the Rule, so that 37.9 will be Number 2, &c.

Let the Diameter of a Cylinder be 37.9 which will be equal to Number 2 on the Rule.

Diameter

Diameter = 37.9

37.9

3411

2653

1137

359) 1436.41 (4.00 Area

0041

Diam. 2 on the new Rule

2

= 37.9 Inch.

4 Equal

In all other Cases of
Circles it infallibly must
answer.

I shall give one Example of Malt, and I think that will make it clear to any unprejudic'd Person, that the Rule is both exact and easie.

The Square Inches allow'd by Statute for a Bushel is 2150. The Root of this Number is 46.36 which is the side of a Square that holds a Bushel of Malt upon an Inch in depth, therefore that is made Number 1 on the Rule, which is placed exactly opposite to 46.36 Inches, and that divided into Decimal and Centesimal parts as before, performs the same work as the Line of Inches, and the set Divisor 2150 doth.

Example, Take the length and breadth of a Floor of Malt by a Line or Tape, and you find the length to be 370.8, and the breadth 185.4, apply this Tape to your new divided Rule, and the length on the new Divisions will be 8, and breadth 4. So that if you had taken the length and breadth of the Floor with any Line, without any regard to Inches, but applying the Line to these new Divisions, you would have found breadth 4 and length 8, which being multiplyed makes 32. equal to the Area of the Floor of Malt at once. But we shall work it by Inches, and see what difference there are.

Length = 370.8

Breadth = 185.4

8 = Length

4 = Breadth

32 Area.

14832

18540

29664

3708

2150) 68746.32 (32.27

4246

20963

15132

082

The Reason of this
being .03 short is be-
cause I threw away
some Decimals for
brevity's sake in wor-
king.

The Rule and Tape with those Divisions are Sold only by Mr. William Haddon, Instrument Maker, in Gilt-spur-street near Newgate, who has the Original Standard from my Projection, and none other but he.

